Supplement to

Capital Mobility and Asset Pricing

Additional Appendices

Darrell Duffie
Stanford University

Bruno Strulovici
Northwestern University

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For simplicity, this supplement uses appendix and equation numbering that continue from the main text of the paper.

D An Insurance Example

We illustrate the model with an example motivated by catastrophe insurance contracts.

In a particular market, at each of the event times of a Poisson process J with a constant intensity η, a catastrophe occurs that causes losses throughout a population of consumers who are potential buyers of protection. Each of a continuum of consumers in the given insurance market has a property that experiences a loss at each catastrophe event. The losses of the consumers at a given event are identically and symmetrically distributed. The distribution of consumer losses at each catastrophe has the property that if a quantity x of the consumers have bought insurance at the time of the i-th catastrophe, then total claims of $x\zeta_i$ are paid by sellers of protection, where $\zeta_1, \zeta_2, \ldots$ is a sequence of independent random variables, identically distributed on $[0, 1]$, and independent of J. For this, it need not be the case that the damage of a particular consumer at the i-th event is equal to the average damage rate ζ_i, but we will assume so for notational simplicity only.

Each consumer chooses to be insured, or not, at each point in time, based on information available up to that time, but of course not including the information about loss events at precisely that time.[22] Whenever insured, the consumer pays premiums at the current rate p_t in his or her market, and is covered against damages in the event of a loss.

[22]The appropriate measurability restriction is "predictability."

Consumer α in a particular market has an insurance purchase policy process δ, valued in $\{0, 1\}$, providing total expected dis-utility of

$$E\left[\int_0^\infty e^{-\beta t} u_{1\alpha}(\delta_t p_t)\, dt + \sum_{i=1}^\infty e^{-\beta \tau_i} u_{2\alpha}((1-\delta_t)\zeta_i)\right],$$

where τ_i is the time of the i-th catastrophe, β is a discount rate, and $u_{1\alpha}(\,\cdot\,)$ and $u_{2\alpha}(\,\cdot\,)$ are strictly decreasing dis-utility functions.

Given the additive nature of this utility, the insurance purchase policy δ minimizes total lifetime dis-utility if and only if, almost everywhere, δ_t solves, time by time, the insurance purchase decision

$$\min_{\overline{\delta} \in \{0,1\}} \; u_{1\alpha}(\overline{\delta} p_t) + \eta E[u_{2\alpha}((1-\overline{\delta})\zeta_i)].$$

This problem is solved by 0 or 1 depending on whether p_t is greater or less than some reservation price p_α. We can therefore calculate, for each premium level $\overline{p}$, the total demand $\chi(\overline{p}) = M\left(\{\alpha : p_\alpha \leq \overline{p}\}\right)$ for insurance, where $M(\,\cdot\,)$ is the measure on the space A of consumers in the market. For example, we can take the set of consumers to be $[0, \infty)$ with Lebesgue measure. Associated with the strictly decreasing demand function χ, assuming continuity, is a strictly decreasing and continuous inverse demand function $\pi(\,\cdot\,)$. That is, $\chi(\pi(x)) = x$. The expected loss rate is $\eta E(\zeta_i)$, so the net risk premium over the risk-free rate r is $\pi(x) - \eta E(\zeta_i) - r$. Provided this risk premium is strictly positive, risk-neutral insurers provide their capital inelastically, unless they have the chance to move their capital to another market. Alternative approaches, for example partial coverage, could be used to model the inverse demand function. In the end, to achieve a tractable solution of the intermediary's problem, we will make parametric assumptions for $\pi(\,\cdot\,)$ that can be justified by suitable construction of $u_{1\alpha}, u_{2\alpha}$, and the measure M.

The cumulative insurance claims process L for a quantity of one unit of insurance sold at all times is the compound Poisson is defined by $L_t = \sum_{i=1}^{J(t)} \zeta_i$. In order to offer one unit of insurance in a particular market, a seller of protection is required to commit one unit of capital. This is natural if one requires (say, as a regulatory matter) that insurance is default free, under the assumption that the essential supremum of the fractional event loss ζ_i is 1, which is the case in our illustrative numerical examples. (In any case, this supremum loss can be taken to be 1 without loss of generality by normalization of the definition of one unit of capital and of the associated construction of returns per unit of capital.)

Markets a and b are assumed to have identically distributed preferences among their respective pools of buyers of protection, and thus have the same inverse-demand function $\pi(\,\cdot\,)$. Their cumulative proportional claims processes L_a and L_b are identically distributed, but need not be independent. For example, some of the loss events could strike both markets.

While capital is deployed in insurance market i, it is subject to the cumulative proportional loss process L_i and is re-invested over time in a financial asset with Lévy cumulative return process R_i. Investment in this additional local asset is allowed merely for generality.

The total cumulative proportional accumulation process for capital in market i, before considering the movement of capital between the markets, is thus $\rho_i = -L_i + R_i$, where ρ_a and ρ_b have the joint distribution described earlier for the general model. Given the characteristics $(q, c, \overline{\lambda})$ of the intermediation of capital between the two markets, the primitives $(\pi, \rho_a, \rho_b, r, q, c, \overline{\lambda})$ of our basic model are fixed.

E Results Supporting the Basic Model

E.1 Homogeneous Case

Allowing somewhat more generality than in the main text, we take the inverse demand function $\pi(\,\cdot\,)$ to be of the form $k_0 + kx^{-\gamma}$ for positive constants k_0, k, and γ. Also without loss of generality, in the following we take $k_0 = 0$ and, by re-scaling, we take $k = 1$. That is, the equilibrium behavior for (k, c) is the same as that for $(1, c/k)$. Because the intermediary has linear time-additive preferences and because of the homogeneity of π, and therefore of ϕ^λ, the ratio $Z = X/Y$ of total capital in the over-capitalized market to total capital in the under-capitalized market determines the optimal intermediation intensity. Thus, we can further assume the independence of ρ_a and ρ_b without loss of generality because any common Lévy component would have no effect on the ratio of X to Y. (The sole exception is a case of common jumps with a jump-size distribution that supports -1, in which case there is a non-zero probability that X_t and Y_t can be zero simultaneously. We rule out this exception.)

Consistent with the insurance example, we suppose that ρ_a and ρ_b are of the form $\rho_{it} = \mu t + \epsilon_{it}$, where μ is a constant and ϵ_a and ϵ_b are independent compound Poisson processes with common jump intensity η and a given jump-size probability distribution

ν. The proportional payoff processes processes ρ_a and ρ_b could also be given a common Brownian component without affecting our analysis, for this also has no effect on the relative proportions of capital in the two markets. Cases with market-specific Brownian components are analyzed in Appendix L. Likewise, the constant drift rate μ plays no role in the analysis of optimal intermediation, and can be taken to be zero without loss of generality for purposes of determining equilibrium intermediation policies. The effect of non-zero μ on actual capital levels can be reintroduced later with the scaling by $e^{\mu t}$ of both X_t and Y_t.

The marginal gain from switching capital is

$$\phi_t^\lambda = F^\Lambda(X_t, Y_t) \equiv H(X_t, Y_t) - G(X_t, Y_t), \tag{36}$$

where, under our regularity, H and G satisfy the coupled equations (1)-(2). For general γ, letting $f(z) = F^\Lambda(z, 1)$ and $L(z) = \Lambda(z, 1)$, the ODE (6) for f generalizes to

$$(r + 2\eta + L(z)(\gamma z + (1 - q)))\, f(z) + z(1 + z)L(z)f'(z) = (1 + \eta g_0)(1 - z^{-\gamma}). \tag{37}$$

E.2 Verification of Optimality of HJB Solution

This appendix provides a proof that the HJB equation (4) characterizes optimality, allowing for a general gain function F^Γ. For this, given an arbitrary intensity process λ, let

$$S_t = e^{-rt}\hat{V}(X_t^\lambda, Y_t^\lambda) + \int_0^t e^{-rs}\lambda_s[X_s^\lambda q F^\Gamma(X_s^\lambda, Y_s^\lambda) - c]\, ds.$$

By Itô's Formula, a local martingale is defined by

$$\hat{V}(X_t^\lambda, Y_t^\lambda) - \int_0^t w(s)\, ds.$$

where

$$w_s = -\hat{V}_x(X_s^\lambda, Y_s^\lambda)\lambda_s X_s^\lambda + \hat{V}_y(X_s^\lambda, Y_s^\lambda)\lambda_s X_s^\lambda + \eta[\hat{V}(X_s^\lambda, 0) + \hat{V}(X_s^\lambda, 0) - 2\hat{V}(X_s^\lambda, Y_s^\lambda)].$$

Because λ and $\hat{V}$ are bounded, this local martingale is in fact a martingale. From this and the implication of the HJB equation that

$$-r\hat{V}(X_t^\lambda, Y_t^\lambda) - \mathcal{U}(\hat{V}, X_t^\lambda, Y_t^\lambda, \lambda_t, \Gamma) \leq 0,$$

another application of Itô's formula implies that S is the sum of a decreasing process and a martingale. Thus, S is a supermartingale. Because $\hat{V}$ is bounded, we have the "transversality" condition that for any intermediation intensity process λ,

$$\lim_{t\to\infty} E[e^{-rt}\hat{V}(X_t^\lambda, Y_t^\lambda)] = 0. \tag{38}$$

Thus, for any intermediation intensity process λ,

$$\hat{V}(x,y) \geq \mathcal{V}(x,y,\lambda,\Gamma) \equiv E\left(\int_0^\infty e^{-rt}\lambda_t[X_t^\lambda q F^\Gamma(X_t^\lambda, Y_t^\lambda) - c]\, dt\right). \tag{39}$$

Let Λ be a policy such that, for each (x,y), $\Lambda(x,y)$ attains the supremum (4). For each t, let $\lambda_t^* = \Lambda(X_t, Y_t)$. Then, the fact that

$$-r\hat{V}(X_t, Y_t) - \mathcal{U}(\hat{V}, X_t, Y_t, \lambda_t^*, \Gamma) = 0$$

implies that S is a martingale. Thus

$$\hat{V}(x,y) = \mathcal{V}(x,y,\lambda^*,\Gamma). \tag{40}$$

Thus, for any intermediation intensity process λ,

$$\hat{V}(x,y) = \mathcal{V}(x,y,\lambda^*,\Gamma) \geq \mathcal{V}(x,y,\lambda,\Gamma),$$

proving the result.

F Algorithm for Trigger Calculation

In general, (16) provides the following fixed-point algorithm for computing the equilibrium trigger capital ratio T.

Combining (14), the equation obtained by differentiating (14), as well as the equation (10) for f, yields the second-order linear ordinary differential equation for v:

$$\alpha v(z) + (\beta + 2z)z(1+z)v'(z) + z^2(1+z)^2 v''(z) = \omega + \delta z, \quad z \geq T, \tag{41}$$

where $\alpha = (a-1)\kappa$, $\beta = (a+\kappa)$, $\omega = d(a-1) - qb$, and $\delta = qb$. We bear in mind that some of the coefficients of this equation depend on a constant to be determined, $v_0 = V(1,0)$.

1. Start with some candidate value for v_0, which we call v^0 (v_0 is bounded above by $2/r$, from the conservation equation). From (17) and (16) we can then determine values for g_0 and T (it is easy to show that such values always exist). Call T^0 the corresponding trigger level. Furthermore, (13) provides a corresponding value for $v(T^0)$.
2. Starting with the initial conditions $v(T^0)$ and $v'(T^0) = 0$, evaluate a candidate for $v(\infty) = \lim_{z\to\infty} v(z)$ by integration of the differential equation (41) on $[T^0, \infty)$.
3. The limit $v(\infty)$ corresponds to a new value for v_0 (since $v(\infty) = V(1,0) = v_0$), which we call v^1.
4. These steps are iterated until a fixed point is reached.

We have considered methods for speeding up the computation.[23]

G Comparative Statics

We now provide proofs of the comparative statics provided in the main text.

G.1 Equilibrium Equations

The optimal threshold T is determined by two conditions. The first is the indifference condition for the intermediary (marginal benefit from switching equals marginal cost).

$$qTf(T) = c. \tag{42}$$

The second equation gives the gain f of switching capital when z is exactly at the threshold:

$$f(T) = \left(1 - \frac{1}{T}\right) \frac{1 + \eta g_0}{r + 2\eta}, \tag{43}$$

[23]One can prove that $v' \sim \log(z)/z^2$ as z goes to ∞. Unfortunately, this convergence rate is not particularly fast. A possible improvement is to integrate v numerically up to some value $\hat{z}$ above which non-dominant terms in (41) are neglected. Above $\hat{z}$, the simplified equation becomes $2z^2v'(z) + z^3v''(z) = 0$, which implies that $v'(z) = v'(\hat{z}) + \log(z)/z^2 - \log(\hat{z})/(\hat{z})^2$. This can be integrated to yield $v(z) - v(\hat{z})$ in closed form (up to the simplification of the equation), since $d(-(\log(z) + 1)/z)/dz = \log(z)/z^2$.

where $g_0 = G(1,0)$ is the aggregate present value of all investors' payoffs after a catastrophe ($X = 1, Y = 0$). Taken together these equations imply that

$$T - 1 = \frac{c}{q}\frac{r + 2\eta}{1 + \eta g_0}. \tag{44}$$

In particular, T is decreasing in g_0. Intuitively, a higher g_0 implies a higher differential in the continuation value, conditional on a shock occurring, between the small and the large markets.

G.2 Effect of Search Cost

First, we examine the effect of search costs.

Proposition 14 *As the search-cost coefficient c increases, so does the associated capital-ratio trigger T.*

Letting $g_0(T)$ denote investors' aggregate payoff when the intermediary uses threshold T, (44) may be rewritten as

$$T - 1 = b(T, c), \tag{45}$$

where

$$b(T, c) = \frac{c}{q}\frac{r + 2\eta}{1 + \eta g_0(T)}.$$

Importantly, changing c has no impact on $g_0(T)$, since the cost c is entirely born by the intermediary, and does not affect the fees paid by investors.[24]

We have seen that, for each c, Equation (45) has a unique root, denoted $T(c)$. Moreover, for each c, $b(\,\cdot\,, c)$ crosses the function $T \mapsto T - 1$ from above on the domain $[1, \infty)$. (Indeed, b is positive, while $T - 1$ starts at 0.)

For each T, the function $b(T, c)$ is increasing in c. Therefore, the solution $T(c)$ must increase in c. As observed earlier, $b(T, c_1) > T - 1$ for all $T < T(c_1)$. Since b is increasing in c, this implies that $b(T, c_2) > T - 1$ for all $T < T(c_1)$. Therefore $T(c_2)$ is greater than $T(c_1)$, as claimed.

[24] This makes the analysis of the effect of c on T simpler than the comparatives statics for other parameters to follow.

G.3 Effect of Fractional Intermediation Fee

Comparative statics for other parameters are more complex because the investor value functions, and thus the intermediation fee function f, depend on them, through g_0.

In order to show dependence on q, we write $T(q)$ for T and $g_0(T, q)$ for g_0. From (44), $T(q)$ is decreasing in q if (for fixed T) $q(1 + \eta g_0(q, T))$ is increasing in q.

Suppose that there exists $q_1 < q_2$ such that $q_1(1 + \eta g_0(q_1, T)) > q_2(1 + \eta g_0(q_2, T))$. From (43), this implies that $q_1 f_1(T) > q_2 f_2(T)$, using the obvious notation. Let

$$\phi(z) = q_2 f_2(z) - q_1 f_1(z).$$

We have $\phi(T) < 0$. Recall the differential equation for f, for $z > T$:

$$\left(\frac{r + 2\eta}{\lambda} + (1 - q) + z\right) f(z) + z(1 + z) f'(z) = \left(1 - \frac{1}{z}\right) \frac{1 + \eta g_0}{\lambda}. \tag{46}$$

Multiplying this ODE for f_2 by q_2, and subtracting from this the product of q_1 and the ODE for f_1, yields

$$\begin{aligned} &\left(\frac{r + 2\eta}{\lambda} + q_1(1 - q_1) + z\right) \phi(z) + z(1 + z)\phi'(z) \\ &= \frac{1}{\lambda}\left(1 - \frac{1}{z}\right)\left(q_2(1 + \eta g_0(q_2, T)) - q_1(1 + \eta g_0(q_1, T))\right) - \left(q_2(1 - q_2) - q_1(1 - q_1)\right) f_2(z). \end{aligned}$$

By assumption, the first term of the right-hand side is negative. Moreover, the function $q \mapsto q(1 - q)$ is increasing on $[0, 1/2]$. Therefore, the right-hand side is negative, provided that $q_2 \leq \frac{1}{2}$. As a result, ϕ cannot cross 0 from below, because at $\phi(z) = 0$ we have $\phi'(z) < 0$. Since $\phi(T) < 0$, we conclude that $\phi(z) < 0$ for all $z \geq T$. That is,

$$q_1 f_1(z) > q_2 f_2(z), \quad z \geq T. \tag{47}$$

From the conservation equation (29), for $i \in \{1, 2\}$,

$$g_0(q_i, T) = \frac{2}{r} - E\left[\int_0^\infty e^{-rt} L(Z_t) q_i Z_t f_i(Z_t)\, dt\right].$$

For a given T, the path of Z_t does not depend on q. Therefore (47) implies that $g_0(q_1, T) < g_0(q_2, T)$. Consequently, $q_1(1 + \eta g_0(q_1, T)) < q_2(1 + \eta g_0(q_2, T))$. This establishes the following result.

Proposition 15 *As q varies in $[0, 1/2]$, the optimal trigger ratio T is decreasing in q.*

The result may be understood as follows: A higher q means a higher fee for the intermediary, other things equal. This prompts the intermediary to search more and, hence, set a lower threshold. However, this intuition is only partially correct, owing to the indirect effect of q through the values of investors. Increasing q lowers the value of investors, lowering the gain to be shared with intermediaries. The previous proof shows that this indirect loss does not offset the direct gain to the intermediary of increasing q. One may show, by a similar argument, the following result, which supports this intuition.

PROPOSITION 16 *For* $q_1 < q_2$*, we have* $g_0(q_1, T(q_1)) > g_0(q_2, T(q_2))$ *and* $f_1(z) > f_2(z)$ *for all* $z > 1$*.*

That result says that a higher fee reduces investors' overall expected payoffs, controlling for equilibrium effects, and reduces the gain from switching capital, at any time, from the larger market to the smaller one.

G.4 Effect of Search Capacity

The impact of $\overline{\lambda}$ on mobility is subtle. A higher capacity means more mobility, as long as the intermediary does switch capital at full capacity. There may, however, be situations for which a low-capacity intermediary switches capital, while a high-capacity does not. We must determine how the optimal threshold T varies with $\overline{\lambda}$.

One might think, intuitively, that a higher capacity reduces the value of switching capital for investors, since they anticipate future switching, and hence, lower heterogeneity. However, that observation does *not* have direct implications for threshold determination. Indeed, by the very definition of the threshold, the intermediary will stop searching after reaching that threshold, no matter what his capacity is. What matters, to compute the gain of switching capital at the threshold, is investors' continuation value after the intermediary stops searching. That continuation value is increasing in g_0 (from (43)). The problem thus reduces to understanding how g_0 varies with search capacity, for fixed T.

Recall, from the conservation equation, that g_0 is decreasing in the present value $E\left[\int_0^\infty e^{-rt} qL(Z_t) f(Z_t)\, dt\right]$ of fees paid to the intermediary.

With a higher search capacity, one might conjecture that the gain from switching, $f(z)$, is lower. However, the heterogeneity level z itself depends on the search capacity parameter $\overline{\lambda}$. Further, as we change the capacity from $\overline{\lambda}_1$ to $\overline{\lambda}_2$, the associated intermediation profit rates $\overline{\lambda}_1 Z_1(t) f_1(Z_1(t))$ and $\overline{\lambda}_2 Z_2(t) f_2(Z_2(t))$ also depend directly on $\overline{\lambda}$. Even if it were true that $f_1(z) > f_2(z)$ for all z, one must consider the dependence of these profit rates directly on $\overline{\lambda}_i$ and $Z_i(t)$.

We will now prove, and provide supporting examples for, the fact that, depending on the discount rate r, the trigger capital ratio T can increase or decrease with capacity.

The proof strategy is as follows, and based on continuity in η for strictly positive $\eta \simeq 0$. First, we fix π and then (artificially), take $\eta = 0$, meaning that there are no shocks. In that case the intermediary switches capital until the threshold is reached, and nothing happens afterwards. The trigger ratio for this case is simply

$$T = 1 + \frac{cr}{q}, \tag{48}$$

which is *independent* of capacity. Moreover, f can be computed independently of g_0 as

$$f(T) = \frac{1}{r}\left(1 - \frac{1}{T}\right)$$

and the ODE for f is

$$(r + ((1-q) + z)\lambda) f(z) + z(1+z)\lambda f'(z) = \left(1 - \frac{1}{z}\right)$$

for $z > T$. The present value of intermediation fees is

$$\int_0^\tau e^{-rt} q\overline{\lambda} Z_t f(Z_t)\, dt,$$

where $\tau = \inf\{t : Z_t = T\}$.

We compute numerically the fees, for different value of $\overline{\lambda}$, and show that g_0 can increase or decrease with $\overline{\lambda}$, depending on r. Specifically, when the discount rate r is low, g_0 is increasing in $\overline{\lambda}$, while when r is high enough, g_0 decreases with $\overline{\lambda}$. Taking this fact as given for now, we conclude that, for η strictly positive but small, the monotonicity of T with respect to $\overline{\lambda}$ depends on r. For this, we let $g_0(\overline{\lambda}, \eta)$ denote the investor-value coefficient associated with search capacity $\overline{\lambda}$ and loss-event intensity η.

Lemma 3 *Fixing $\pi(\cdot)$, suppose that for some $\overline{\lambda}_1 \neq \overline{\lambda}_2$ we have $g_0(\overline{\lambda}_1, 0) < g_0(\overline{\lambda}_2, 0)$. Then there exists some $\bar{\eta} > 0$ such that the optimal respective trigger ratios T_1 and T_2 satisfy $T_1 > T_2$ for all $\eta \in (0, \bar{\eta})$.*

Proof. By continuity, $g_0(\overline{\lambda}_1, \eta) < g_0(\overline{\lambda}_2, \eta)$ for all η less than some $\bar{\eta} > 0$. The result then follows from (44).

G.5 Effect of Shock Frequency

We now show that the optimal threshold T is increasing in η, holding the dividend payout function π fixed. We use the fact that T solves the equation

$$T - 1 = b(T, \eta)$$

where

$$b(T, \eta) = \frac{c}{q} \frac{r + 2\eta}{1 + \eta g_0(\eta, T)}.$$

Proceeding as for the case of c, it is enough to show that $b(T, \eta)$ is increasing in η.

Suppose, for purposes of producing a contradiction, that $\eta_1 < \eta_2$ but, fixing T,

$$\frac{r + 2\eta_1}{1 + \eta_1 g_0(\eta_1, T)} > \frac{r + 2\eta_2}{1 + \eta_2 g_0(\eta_2, T)}.$$

This implies that $f_1(T) < f_2(T)$, as indicated by (43). We will show that $f_1(z) < f_2(z)$ for all z. Let $\phi(z) = f_2(z) - f_1(z)$. By assumption, $\phi(T) > 0$. We divide the dynamic equation for f_2 (see (46)) by $r + 2\eta_2$ and subtract from it the dynamic equation for f_1 divided by $r + 2\eta_1$. The result, after rearranging, is

$$\begin{aligned}
\left(1 + \frac{\lambda}{2 + \eta_2}((1-q) + z)\right)\phi(z) + \frac{\lambda}{r + 2\eta_2} z(1+z)\phi'(z)& \\
= \left(1 - \frac{1}{z}\right)\left(\frac{1 + \eta_2 g_0(\eta_2, T)}{r + 2\eta_2} - \frac{1 + \eta_1 g_0(\eta_1, T)}{r + 2\eta_1}\right)& \\
+ \left(\frac{\lambda}{r + 2\eta_1} - \frac{\lambda}{r + 2\eta_2}\right)\left(((1-q) + z) f_1(z) + z(1+z) f_1'(z)\right).& \quad (49)
\end{aligned}$$

The right-hand side is positive. Indeed, its first term is positive by assumption, and the second term is positive because, from (46), the second factor is equal to

$$\left(1 - \frac{1}{z}\right)(1 + \eta g_0) - (r + 2\eta) f_1(z) = (r + 2\eta)\varrho_1(z),$$

which is non-negative because of Lemma 1. This implies that if $\phi(z) = 0$, then $\phi'(z) > 0$, so that ϕ cannot cross zero from above. Since $\phi(T)$ is positive, we conclude that ϕ is everywhere positive, so $f_2(z) > f_1(z)$.

We now build a contradiction. The fact that $f_1(T) < f_2(T)$ implies that $g_0(\eta_1, T) < g_0(\eta_2, T)$. Indeed, $(1 + \eta g)/(r + 2\eta)$ is increasing in g and decreasing in η (since $g < 2/r$, from the conservation equation). Therefore, if $\eta_2 > \eta_1$, we can have

$$\frac{1 + \eta_2 g_0(\eta_2, T)}{r + 2\eta_2} > \frac{1 + \eta_1 g_0(\eta_1, T)}{r + 2\eta_1}$$

only if $g_0(\eta_2, T) > g_0(\eta_1, T)$. Proceeding as in Section G.3, we now show that $f_2(z) > f_1(z)$ implies that $g_0(\eta_2, T) < g_0(\eta_1, T)$, which will yield the contradiction. From the conservation equation, it suffices to show that

$$E\left[\int_0^\infty e^{-rt} Z_1(t) f_1(Z_1(t))\, dt\right] < E\left[\int_0^\infty e^{-rt} Z_2(t) f_2(Z_2(t))\, dt,\right]$$

where the subscripts indicate the role of η. (Under η_2, loss events are more frequent in expectation.) Let

$$\alpha_i = \frac{E[\int_0^{\tau_i} e^{-rt} Z_i(t) f_i(Z_i(t))\, dt]}{E[\int_0^{\tau_i} e^{-rt}\, dt]},$$

where τ^i is the time of the first jump of Z_i. To ease the comparison, we can without loss of generality choose the probability space so that $\tau_1 > \tau_2$ almost surely. Because (i) $zf(z)$ is increasing in z, (ii) Z_t is decreasing between jumps, (iii) $\tau_1 > \tau_2$, and (iv) $f_1(z) < f_2(z)$, we conclude that

$$\alpha_1 < \alpha_2.$$

Let $\delta_i = E[e^{-r\tau_i}]$. One may easily check, breaking down the integral by loss event times, that

$$E\left[\int_0^\infty e^{-rt} Z_i(t) f_i(Z_i(t))\, dt\right] = \sum_{k\geq 0} \delta_i^k \left(\alpha_i \frac{1-\delta_i}{r}\right) = \frac{\alpha_i}{r},$$

where the second factor in each sum is the present value of the cash flows between any two consecutive loss events, which is constant, given the stationarity of the problem. Since $\alpha_1 < \alpha_2$, this yields the desired inequality, and concludes the proof.

Proposition 17 *Fixing $\pi(\,\cdot\,)$, the optimal trigger capital ratio T is increasing in the mean frequency η of loss events.*

G.6 Effect of Discount Rate

Finally, we show that T is increasing with r. It is immediately clear why monotonicity should obtain: a higher r means a lower gain from switching, other things equal, since future cash flows are discounted more. However, this reduces the payments to the intermediary and could a priori result in a higher continuation value g_0, and hence a lower threshold. Again, we need to show that the indirect effect is dominated by the direct one.

Suppose not, and for some $r_1 < r_2$ the associated optimal trigger ratios satisfy $T_1 > T_2$. Letting $T = T_1$, we have

$$f_1(T) < f_2(T),$$

from (42) and the fact that $z \mapsto zf(z)$ is increasing in z. From (44), we also have

$$\frac{r_1 + 2\eta}{1 + \eta g_0(r_1, T)} > \frac{r_2 + 2\eta}{1 + \eta g_0(r_2, T)}. \tag{50}$$

Since $r_1 < r_2$, (50) also implies that

$$g_0(r_1, T) < g_0(r_2, T). \tag{51}$$

We now show that $f_1(z) < f_2(z)$ for all $z > T$. Let $\phi(z) = f_2(z) - f_1(z)$. Then, $\phi(T) > 0$. Taking the difference of the equations (46) for r_2 and r_1, and rearranging, we get

$$\begin{aligned}\overline{\lambda}((1-q)+z)\phi(z) + \overline{\lambda}z(1+z)\phi'(z) &= \left(1 - \frac{1}{z}\right)[(1 + \eta g_0(r_2, T)) - (1 + \eta g_0(r_1, T))] \\ &\quad -(r_2 + 2\eta) f_2(z) + (r_1 + 2\eta) f_1(z). \end{aligned} \tag{52}$$

Suppose that $\phi(z) = 0$ for some $z > T$, that is, $f_2(z) = f_1(z)$. We now show that this implies that the right-hand side of (52) is positive. From (50), and the fact that $f_1(z) = f_2(z)$, we have

$$\left(1 - \frac{1}{z}\right)\left(\frac{1 + \eta g_0(r_1, T)}{r_1 + 2\eta}\right) - f_1(z) < \left(1 - \frac{1}{z}\right)\left(\frac{1 + \eta g_0(r_2, T)}{r_2 + 2\eta}\right) - f_2(z) = \varrho_2(z).$$

Because Lemma 1 implies that $\varrho_2(z) \geq 0$, the right-hand side of (52) is greater than

$$(r_2 - r_1)\varrho_2(z) \geq 0.$$

This implies that $\phi'(z) \geq 0$ and, hence, that $\phi(z)$ cannot cross zero from above, which proves the claim that $f_1(z) < f_2(z)$ for all $z > T$.

Since $T_1 > T_2$, this implies that

$$L_1(Z_1(t))Z_1(t)f_1(Z_1(t)) \leq L_2(Z_2(t))Z_2(t)f_2(Z_2(t))$$

for all t. Indeed, immediately after a shock, the paths of Z_1 and Z_2 are identical until, possibly, they reach T_1, after which $L_1(Z_1(t)) = 0$. From the conservation equation, for $i \in \{1, 2\}$,

$$g_0(r_i, T) = E\left[\int_0^\infty e^{-r_i t}\left(2 - qL_i(Z_i(t))Z_i(t)f_i(Z_i(t))\right)\, dt\right].$$

Let $\chi_t = E[2 - qL_2(Z_2(t))Z_2(t)f_2(Z_2(t))]$ and

$$\xi_{it} = \int_t^\infty e^{-r_i s}\chi_s\, ds.$$

We have $\xi_{20} = g_0(r_2, T)$ and $\xi_{2t} > 0$ for all t. The first claim is obvious. For the second claim, observe that

$$e^{r_2 t}\xi_{2t} = E\left[E\left[\int_t^\infty e^{-r_2(s-t)}\left(2 - qL_2(Z_2(s))Z_2(s)f_2(Z_2(s))\right)\, ds \;\middle|\; Z_2(t)\right]\right].$$

The inner conditional expectation is the expected payoff aggregated over all investors from time t onwards. Since each investor gets a positive profit (he can always stay in a given market and pay no fee, and get a positive payoff), that aggregate profit is positive. We now use the following lemma, found in Quah and Strulovici (2009) and Quah and Strulovici (2010) and proved using an integration by parts.

LEMMA 4 *Suppose that γ and h are integrable real-valued functions defined on $[0,\infty)$, with γ increasing. If $\int_t^\infty h(s)\, ds \geq 0$ for all $t \geq 0$, then*

$$\int_0^\infty \gamma(s)h(s)\, ds \geq \gamma(0)\int_0^\infty h(s)\, ds. \tag{53}$$

Applying the lemma to $h(s) = \chi_s$ and $\gamma(s) = e^{(r_2 - r_1)s}$, we conclude that

$$\int_0^\infty e^{-r_1 t}\chi_t\, dt \geq g_0(r_2, T).$$

Because we also have $\chi_t < 2 - qE[L_1(Z_1(t))Z_1(t)f_1(Z_1(t))]$ for all t, we conclude that

$$g_0(r_1, T) \geq g_0(r_2, T),$$

which yields the desired contradiction.

H The Case of Partial Recovery

We now allow the fraction W recovered after a loss to be randomly distributed on $(0, 1)$. This will be the basis for our numerical illustration of the model. Subject to the usual smoothness and integrability conditions, Itô's formula and the definition (24) of the value of a unit of capital held in market i imply that the value $G(x, y)$ of a unit of capital in the over-capitalized market satisfies:

$$\begin{aligned} 0 \;=\; & -rG(x,y) + \pi(x) - G_x(x,y)x\Lambda(x,y) + G_y(x,y)x\Lambda(x,y) \\ & + \eta\, P(Wx < y)[E(H(y,xW)\,|\,Wx < y) - G(x,y)] \\ & + \eta P(Wx \geq y)[E(G(Wx,y)\,|\,Wx \geq y) - G(x,y)] \\ & + (1-q)\eta(H(x,y) - G(x,y)) + \eta[E(G(x,Wy) - G(x,y)]. \end{aligned}$$

The analogous equation for H is

$$\begin{aligned} 0 &= -rH(x,y) + \pi(y) - H_x(x,y)x\Lambda(x,y) + H_y(x,y)x\Lambda(x,y) \\ &\quad + \eta P(Wx < y)[E(G(y,Wx) \,|\, Wx < y) - H(x,y)] \\ &\quad + \eta P(Wx \geq y)[E(H(Wx,y) \,|\, Wx \geq y) - H(x,y)] \\ &\quad + \eta E[(H(x,Wy) - H(x,y)]. \end{aligned}$$

We let $\Phi(\,\cdot\,)$ denote the cumulative recovery-rate distribution function associated with the fractional event loss measure ν. That is, $\Phi(u) = 1 - \nu([0,u])$. We let $g(z) = G(z,1)$ and $h(z) = H(z,1)$, obtaining the coupled equations

$$\begin{aligned} (r + 2\eta + \Lambda(z,1)z)g(z) + \Lambda(z,1)(1+z)zg'(z) &= \frac{1}{z} + \Lambda(z,1)(1-q)(h(z) - g(z)) \\ &\quad + \eta\left[\int_{1/z}^{1} ug(uz)\,d\Phi_u + \int_0^{1/z} \frac{1}{z} h\left(\frac{1}{uz}\right)\,d\Phi_u + \int_0^1 \frac{1}{u} g\left(\frac{z}{u}\right)\,d\Phi_u\right] \end{aligned} \tag{54}$$

and

$$\begin{aligned} &(r + 2\eta + \Lambda(z,1)z)h(z) + \Lambda(z,1)(1+z)zh'(z) \\ &\qquad = 1 \,+\, \eta\left[\int_{1/z}^{1} h(uz)\,d\Phi_u + \int_0^{1/z} \frac{1}{uz} g\left(\frac{1}{uz}\right)\,d\Phi_u + \int_0^1 h\left(\frac{z}{u}\right)\,d\Phi_u\right]. \end{aligned} \tag{55}$$

As opposed to the case of total loss, these equations cannot be combined to yield a single equation for $f = h - g$, because of differing integrands.

Letting $v(z) = V(z,1)$, the 0-homogeneity of V implies that the value after a loss event is $v(uz)$ if $ux \geq y$, $v(1/uz)$ if $ux \leq y$, and $v(z/u)$ if the loss occurs on the smaller market. The HJB equation is thus

$$\begin{aligned} 0 = \sup_{\ell \in [0,\overline{\lambda}]} \Bigg\{ & -rv(z) - \ell z v'(z) - \ell z^2 v'(z) + \ell(qzf(z) - c) \\ & + \eta\left[\int_{1/z}^{1} v(uz)\,d\Phi_u + \int_0^{1/z} v\left(\frac{1}{uz}\right)\,d\Phi_u + \int_0^1 v\left(\frac{z}{u}\right)\,d\Phi_u - 2v(z)\right]\Bigg\}. \end{aligned} \tag{56}$$

The equation reduces to

$$(r + 2\eta)v(z) = \eta\left[\int_{1/z}^{1} v(uz)\,d\Phi_u + \int_0^{1/z} v\left(\frac{1}{uz}\right)\,d\Phi_u + \int_0^1 v\left(\frac{z}{u}\right)\,d\Phi_u\right], \quad z \in [1,T],$$

and

$$(r+2\eta)v(z)+\overline{\lambda}(1+z)zv(z)' = [qzf(z)-c]\overline{\lambda}$$
$$+\eta\left[\int_{1/z}^{1} v(uz)\,d\Phi_u + \int_0^{1/z} v\left(\frac{1}{uz}\right)\,d\Phi_u + \int_0^1 v\left(\frac{z}{u}\right)\,d\Phi_u\right], \quad z \geq T. \quad (57)$$

The smooth-pasting condition is

$$(1+T)Tv'(T) = qTf(T) - c. \quad (58)$$

In this setting, the intermediary's value function cannot be computed by solving a differential equation because $v'(z)$ depends on $v(z')$ for all other z'. We have the same issue to overcome in order to solve for $G(x,y)$ and $H(x,y)$. Exploiting the linear structure of the problem, however, Appendix K provides a numerical algorithm for solving the corresponding integro-differential equations. The associated smooth-fit condition is

$$qTf(T) - c = T(1+T)v'(T). \quad (59)$$

I Numerical Illustration with Partial Recovery

We provide an illustrative example of equilibrium for the case of partial recovery, which is analyzed in Appendix H. We take the parameters $r = 0.04$, $\eta = 1.5$, $c = 0.04$, $\overline{\lambda} = 0.1$, $q = 1/30$. We assume beta-distributed recovery (one minus proportion lost) on $(0,1)$, with parameters $(5,1)$. The equilibrium intermediation trigger ratio T of capital in the over-capitalized market to capital in the under-capitalized market is found numerically to be 1.465.

Figure 1 shows simulated sample paths of the capitalization ratio $Z_t = X_t/Y_t$ and the immediate return $f(Z_t)/g(Z_t)$ to a supplier of capital, before transactions fees, associated with switching capital into the under-capitalized market. Figure 2 shows the present values, with one unit of capital in the under-capitalized market, of future cash flows to a provider of one unit capital in the over-capitalized market (net of fees), to a provider of one unit of capital in the under-capitalized market (net of fees), and to the intermediary (in the form of fees net of search costs). These are, respectively, $g(z)$, $h(z)$, and $v(z)$, and depend on the ratio $z = x/y$ of the level of capital x in the over-capitalized market to the level y of capital in the under-capitalized market.

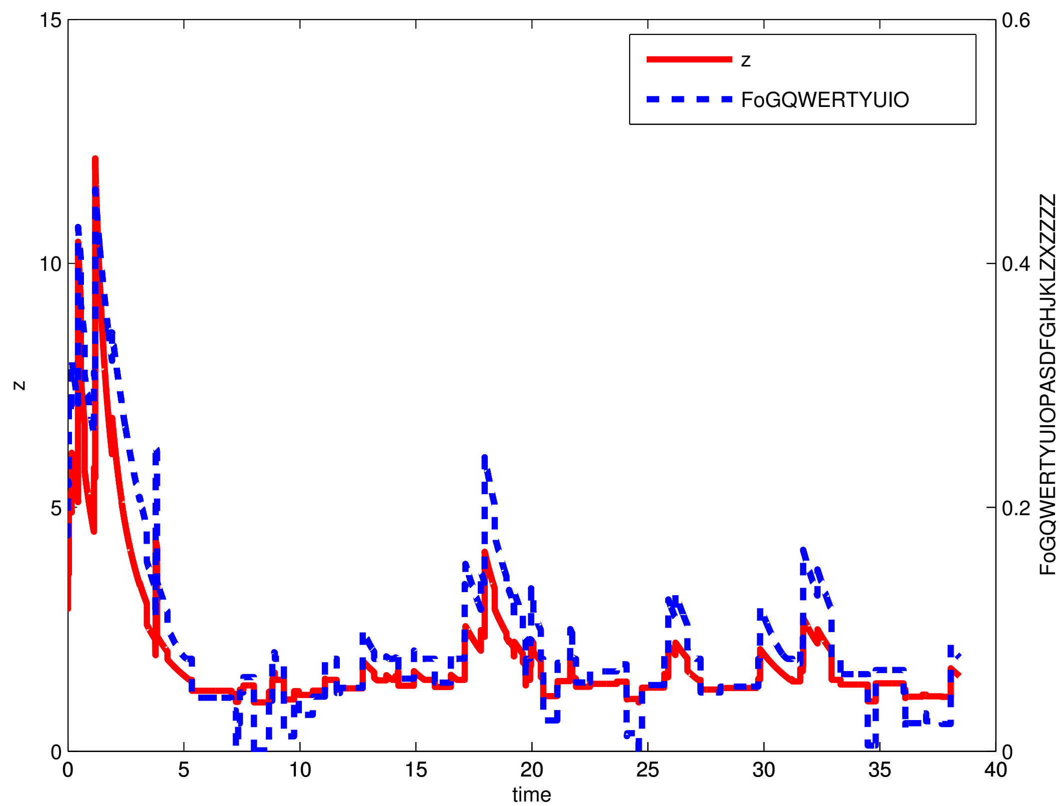

Figure 1: Simulated sample paths of the capitalization ratio, $Z_t = X_t/Y_t$, and the return from switching, $f(Z_t)/g(Z_t)$.

J Intermediary Competition with Partial Recovery

Here, we discuss the case of oligopolistic competition with partial recovery. Recall from (59) the smooth-pasting condition for the monopolistic case:

$$qTf(T) - c = T(1+T)v'(T). \tag{60}$$

One can see that the trigger capital ratio T is determined not only by the function f determining the marginal gain from moving capital, but also by the derivative $v'(T)$ of the intermediary's value function. In order to understand the impact of oligopolistic intermediation, suppose that intermediaries were to use, instead of the optimal trigger ratio T, the equilibrium trigger ratio of a monopolist with the same aggregate capacity for intermediation. In that case, f would be unchanged. Each intermediary, however, would receive only a fraction $1/n$ of the total intermediation fees. The righthand side of (60) is thus lowered, implying that intermediaries prefer to continue intermediating

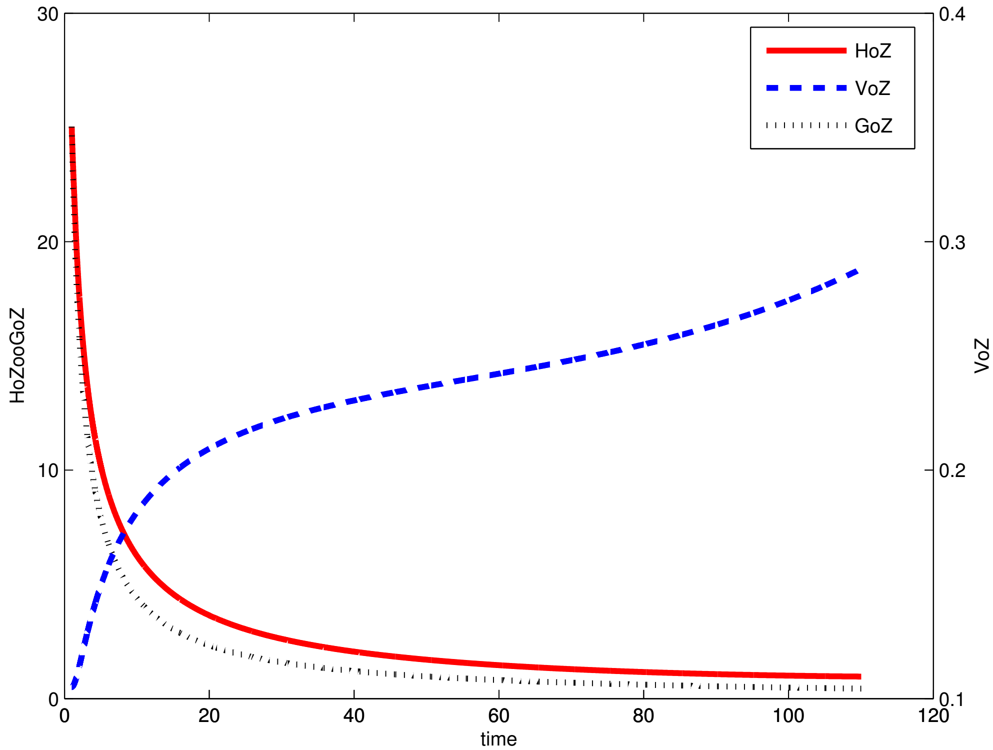

Figure 2: Value function $v(z)$ of the intermediary and the marginal values $g(z)$ and $h(z)$ of capital held in the over-capitalized and undercapitalized markets, respectively.

after the capital ratio exceeds the monopolistic trigger. This is the first channel through which oligopolistic competition matters: Because an oligopolistic intermediary does not internalize the full impact of his search on intermediation fees, he has a greater incentive to intermediate. More precisely, an intermediary does not work for opportunities to move capital when the immediate net marginal benefit of doing so, $qzf(z) - c$, is below the marginal value $z(1+z)v'(z)$ associated with future capital heterogeneity. For a given trigger ratio T, an intermediary's value function v declines in direct proportion to the number n of intermediaries, and, hence, so does the derivative v'. This implies that the term $z(1+z)v'(z)$ diminishes with n, while the immediate marginal benefit $qzf(z) - c$ is unchanged, keeping T constant. Thus, as n increases, the incentive to intermediate at the given trigger ratio T becomes strictly positive, prompting intermediaries to search more.[25]

As n goes to infinity, an intermediary's value function goes to zero (because the size of the pie to be shared among intermediaries is uniformly bounded above by $2/r$), and the derivative $v'(T)$ also goes to 0. The limit as n diverges is the competitive equilibrium, in which the trigger capital ratio T is determined by

$$qTf(T) - c = 0.$$

With perfect competition, an intermediary has no impact on aggregate search activity, and thus cares only about the immediate net benefit from switching.

K Algorithm for the Case of Partial Recovery

This appendix includes an algorithm for solving the partial-recovery version of our model. The algorithm exploits the linearity of the integro-differential equations for g, h, and v, which arise thanks to the special structure of our problem.

K.1 Primitives

The parameters are r, η, $\overline{\lambda}$, q, c, and the recover rate distribution function $\Phi : [0, 1] \to [0, 1]$, a beta distribution with given parameters. The algorithm will determine the trigger level T for intermediation and the value functions g, h, and v.

K.2 Strategy

We use the following fixed-point algorithm. Start with a value of T, then iterate the following steps:

1. Numerically evaluate g and h (which are independent of the rest of the system, given T).
2. Numerically evaluate v (which depends on T, g and h).
3. Use (58) to obtain a new value of T.

[25]When there is zero recovery from a loss event, the after-event heterogeneity (which is infinite) does not depend of the pre-event heterogeneity. In that case, intermediaries already ignore the impact of their search activity on heterogeneity and the monopolistic solution coincides with the competitive one.

4. Stop if the last iteration is such that the new value of T is close enough to the value of T at the beginning of the loop. Otherwise, return to the first step.

Relation (16) also provides an upper bound on the equilibrium capital-ratio trigger level:

$$T \leq 1 + \frac{c(r + 2\eta)}{q}.$$

Thus, the solution T lies in $1 \leq T \leq 1 + c(r + 2\eta)/q$ which bounds the starting value.

The remaining subsections provide guidelines for the realization of each step. Except for the last subsection, the value of T is fixed.

K.3 A system of equations for g and h

We first discretize the equations for g and h to obtain a linear system of equations of the form

$$Ax = b.$$

The variable $z \in [1, \infty)$ is discretized: we use a grid $\mathcal{G}$ with $n+1$ points such that $z_i = \delta^i$, $i \in \{0, \ldots, n\}$, where $\delta > 1$ is fixed. Such a grid is finer near 1, where T is more likely to be found. Considering other grids does not affect the equations below.

To each z_i corresponds two rows of the matrix A, which is $(2n + 2) \times (2n + 2)$. The vector $x = [g, h]$ corresponds to the discretized values of the unknown functions g and h. In what follows, $g = (g_0, \ldots, g_n)$ and $h = (h_0, \ldots, h_n)$ are vectors approximating the functions, and x is the concatenation of these vectors.

For any condition C let 1_C denote the function equal to 1 if C is true and 0 otherwise.

For z and T in $\mathcal{G}$, we let $\lambda(z, T) = \overline{\lambda} 1_{z>T}$. Thus, $\lambda = \overline{\lambda}$ if $z > T$ and 0 otherwise.

K.4 Discretization conventions

For any $0 \leq u < u' \leq 1$, we let $K(u, u') = \Phi(u') - \Phi(u)$ denote the probability that the recovery rate is between u and u', according to the stipulated beta distribution. For each i, let $\lambda_i = \lambda(z_i, T)$

In the computations to follow, we let $z_{-1} = 1$, $z_{n+1} = z_n$, $g_{-1} = g_0$, $g_{n+1} = g_n$, $h_{-1} = h_0$, and $h_{n+1} = h_n$.

K.5 Discretized Equations

The discretized equation for g yields, for $i \in \{0, \ldots, n\}$,

$$\begin{aligned}
g_i[r + 2\eta + \lambda_i(z_i + (1-q))] &+ g_{i+1}\frac{\lambda_i z_i(1+z_i)}{z_{i+1} - z_{i-1}} + g_{i-1}\frac{-\lambda_i z_i(1+z_i)}{z_{i+1} - z_{i-1}} + h_i(q-1)\lambda_i \\
&- \eta \sum_{j=0}^{i} g_j \frac{z_j}{z_i} K\left(\frac{z_{j-1} + z_j}{2z_i}, \frac{z_j + \min\{z_{j+1}, z_i\}}{2z_i}\right) \\
&- \eta \sum_{j=0}^{n} \frac{h_j}{z_i z_j} K\left(1_{j<n}\left(\frac{1}{2z_i z_j} + \frac{1}{2z_i z_{j+1}}\right), \frac{1}{2z_i z_j} + \frac{1}{2z_i z_{j-1}}\right) \\
&- \eta \sum_{j=i}^{n} g_j \frac{z_j}{z_i} K\left(1_{j<n}\left(\frac{z_i}{2z_j} + \frac{z_i}{2z_{j+1}}\right), \frac{z_i}{2z_j} + \frac{z_i}{2\max\{z_{j-1}, z_i\}}\right) = \frac{1}{z_i}. \quad (61)
\end{aligned}$$

The discretized equation for h yields, for $i \in \{0, \ldots, n\}$,

$$\begin{aligned}
h_i[r + 2\eta + \lambda_i z_i] &+ h_{i+1}\frac{\lambda_i z_i(1+z_i)}{z_{i+1} - z_{i-1}} + h_{i-1}\frac{-\lambda_i z_i(1+z_i)}{z_{i+1} - z_{i-1}} \\
&- \eta \sum_{j=0}^{i} h_j K\left(\frac{z_{j-1} + z_j}{2z_i}, \frac{z_j + \min\{z_{j+1}, z_i\}}{2z_i}\right) \\
&- \eta \sum_{j=0}^{n} g_j z_j K\left(1_{j<n}\left(\frac{1}{2z_i z_j} + \frac{1}{2z_i z_{j+1}}\right), \frac{1}{2z_i z_j} + \frac{1}{2z_i z_{j-1}}\right) \\
&- \eta \sum_{j=i}^{n} h_j K\left(1_{j<n}\left(\frac{z_i}{2z_j} + \frac{z_i}{2z_{j+1}}\right), \frac{z_i}{2z_j} + \frac{z_i}{2\max\{z_{j-1}, z_i\}}\right) = 1. \quad (62)
\end{aligned}$$

K.6 Linear system

We index from 0 to $2n+1$ the rows and columns of A as well as the rows of b. Indices from 0 to n correspond to equations or variables related to g, while indices from $n+1$ to $2n+1$ correspond to equations or variables related to h. The above discretized equations determine the coefficients of A and b. First, $b_i = 1/z_i$ for $i \leq n$ and $b_i = 1$ for $i > n$, as is clear from the above. We can decompose A into four $(n+1) \times (n+1)$ submatrices as

$$A = \begin{bmatrix} B & C \\ D & E \end{bmatrix}.$$

The coefficients of these submatrices are determined by the previous discretized equations. We have

$$B_{ii} = r + 2\eta + \lambda_i(z_i + (1-q)) - \eta\left[K\left(\frac{z_{i-1} + z_i}{2z_i}, 1\right) + K\left(\frac{1}{2} + \frac{z_i}{2z_{i+1}}, 1\right)\right].$$

For $i < n$,

$$B_{i(i+1)} = \frac{\lambda_i z_i(1+z_i)}{z_{i+1} - z_{i-1}} - \eta \frac{z_{i+1}}{z_i} K\left(1_{i+1<n}\left(\frac{z_i}{2z_{i+1}} + \frac{z_i}{2z_{i+2}}\right), \frac{z_i}{2z_{i+1}} + \frac{1}{2}\right).$$

For $i > 0$,

$$B_{i(i-1)} = \frac{-\lambda_i z_i(1+z_i)}{z_{i+1} - z_{i-1}} - \eta \frac{z_{i-1}}{z_i} K\left(\frac{z_{i-2} + z_{i-1}}{2z_i}, \frac{z_{i-1} + z_i}{2z_i}\right).$$

For all i and $j > i+1$,

$$B_{ij} = -\eta \frac{z_j}{z_i} K\left(1_{j<n}\left(\frac{z_i}{2z_j} + \frac{z_i}{2z_{j+1}}\right), \frac{z_i}{2z_j} + \frac{z_i}{2\max\{z_{j-1}, z_i\}}\right).$$

For all i and $j < i-1$,

$$B_{ij} = -\eta \frac{z_j}{z_i} K\left(\frac{z_{j-1} + z_j}{2z_i}, \frac{z_j + \min\{z_{j+1}, z_i\}}{2z_i}\right).$$

The coefficients of the matrices C, D, and E can be obtained similarly.

Once A is computed, we solve the system $A[g; h] = b$. This yields the vector of candidate values for g and h that is needed in the next step of the algorithm.

For $n = 100$, the system can easily be solved by any reasonable computation package, as long as A is invertible. Usual algorithms proceed by factorization of A and direct computation of the solution by pivot methods, which are faster and more robust than inversion of A.

K.7 Computation of v

We discretize the equation for v similarly, using the candidate values of g and h obtained in the previous step. The goal of this subsection is to determine the coefficients of the matrix F and a vector d defining the system $Fv = d$, where $v \in \mathbb{R}_+^{n+1}$ is the discretization vector of the function v, F is a $(n+1) \times (n+1)$ square matrix, and d is an $(n+1)$-dimensional vector.

The discretized equation for $v = (v_0, \ldots, v_n)$ yields for $i \in \{0, \ldots n\}$, keeping the same notational scheme used before and, letting $v_{-1} = v_0$ and $v_{n+1} = v_n$,

$$\begin{aligned}
v_i[r+2\eta] + v_{i+1}\frac{\lambda_i z_i(1+z_i)}{z_{i+1}-z_{i-1}} + v_{i-1}\frac{-\lambda_i z_i(1+z_i)}{z_{i+1}-z_{i-1}}& \\
-\eta\sum_{j=0}^{i} v_j K\left(\frac{z_{j-1}+z_j}{2z_i}, \frac{z_j+\min\{z_{j+1},z_i\}}{2z_i}\right)& \\
-\eta\sum_{j=0}^{n} v_j K\left(1_{j<n}\left(\frac{1}{2z_i z_j}+\frac{1}{2z_i z_{j+1}}\right), \frac{1}{2z_i z_j}+\frac{1}{2z_i z_{j-1}}\right)& \\
-\eta\sum_{j=i}^{n} v_j K\left(1_{j<n}\left(\frac{z_i}{2z_j}+\frac{z_i}{2z_{j+1}}\right), \frac{z_i}{2z_j}+\frac{z_i}{2\max\{z_{j-1},z_i\}}\right) &= \lambda_i[qz_i(h_i-g_i)-c]. \quad (63)
\end{aligned}$$

Therefore, the right-hand side of the linear system is $d_i = \lambda_i[qz_i(h_i - g_i) - c]$. The coefficients F are determined as were those of A.

K.8 New Value of T

The last step of the loop of the fixed-point algorithm is the determination of a new candidate trigger level of T. Discretizing (58) yields the condition, for $T = z_t$

$$(1+z_t)z_t\frac{v_{t+1}-v_{t-1}}{z_{t+1}-z_{t-1}} = qz_t(h_t-g_t)-c.$$

The new candidate value of T is thus the element of the grid $\mathcal{G}$ whose corresponding index t is the closest to satisfying the above equation.

L Diffusion Risk

In this appendix, we allow invested capital to be exposed to diffusive reinvestment risk. Specifically, we suppose that the Lévy process ρ_i driving proportional capital changes in market i is the sum of a Brownian motion ζ_i and an independent compound Poisson process. The value function retains the same degree of homogeneity found in the main text.

With perfect correlation between the Brownian sources of risk in the two markets, ζ_a and ζ_b, the analysis is identical to that shown in the main text.

More generally, suppose that the Brownian motions ζ_a and ζ_b have volatility parameter σ and correlation parameter R. In the remainder of this appendix, we derive the characterizing equations for G and H, then g and h.

To clarify computations with diffusion terms, we temporarily consider investor wealth. Let $\tilde{G}(x,y,\alpha)$ and $\tilde{H}(x,y,\alpha)$ denote the present value of having α units of capital initially in the large and small markets, respectively. Of course, $\tilde{G}(x,y,\alpha) = \alpha G(x,y)$, where $G(x,y) = \tilde{G}(x,y,1)$. Similarly, $\tilde{H}(x,y,\alpha) = H(x,y)$, where $H(x,y) = \tilde{H}(x,y,1)$. We first provide equations for $\tilde{G}$ and $\tilde{H}$, and then use those to derive equations for G and H.

We assume, to begin, zero recovery. As before, we can take the drift rate μ to be zero without loss of generality. We have

$$\begin{aligned}
&-r\tilde{G}(x,y,\alpha) + \alpha\pi(x) - \tilde{G}_x(x,y,\alpha)x\Lambda(x,y) + \tilde{G}_y(x,y,\alpha)x\Lambda(x,y) \\
&\quad + (1-q)\eta(\tilde{H}(x,y,\alpha) - \tilde{G}(x,y,\alpha)) - \eta\tilde{G}(x,y,\alpha) + \eta(\tilde{G}(x,0,\alpha) - \tilde{G}(x,y,\alpha)) \\
&\quad + \frac{1}{2}\sigma^2[\tilde{G}_{xx}(x,y,\alpha)x^2 + \tilde{G}_{yy}(x,y,\alpha)y^2 + \tilde{G}_{\alpha\alpha}(x,y,\alpha)\alpha^2] \\
&\quad + \sigma^2[xyR\tilde{G}_{xy}(x,y,\alpha) + x\alpha\tilde{G}_{x\alpha}(x,y,\alpha) + y\alpha R\tilde{G}_{y\alpha}(x,y,\alpha)] = 0
\end{aligned} \tag{64}$$

and

$$\begin{aligned}
&-r\tilde{H}(x,y,\alpha) + \alpha\pi(y) - \tilde{H}_x(x,y,\alpha)x\Lambda(x,y) + \tilde{H}_y(x,y,\alpha)x\Lambda(x,y) \\
&\quad + \eta(\tilde{G}(y,0,\alpha) - \tilde{H}(x,y,\alpha)) - \tilde{\eta}H(x,y,\alpha) \\
&\quad + \frac{1}{2}\sigma^2[\tilde{H}_{xx}(x,y,\alpha)x^2 + \tilde{H}_{yy}(x,y,\alpha)y^2 + \tilde{H}_{\alpha\alpha}(x,y,\alpha)\alpha^2] \\
&\quad + \sigma^2[xyR\tilde{H}_{xy}(x,y,\alpha) + x\alpha R\tilde{H}_{x\alpha}(x,y,\alpha) + y\alpha\tilde{H}_{y\alpha}(x,y,\alpha)] = 0,
\end{aligned} \tag{65}$$

where we used the fact that, when the investor is in market x, the correlation between x and α is 1, and the correlation between y and α is R. The symmetric correlations apply when the investor is in market y.

Using the fact that $\tilde{G}_\alpha(x,y,1) = G(x,y)$, $\tilde{G}_{\alpha\alpha}(x,y,1) = 0$, $\tilde{G}_{x\alpha}(x,y,1) = G_x(x,y)$, and $\tilde{G}_{y\alpha}(x,y,1) = G_y(x,y)$, with identical relations between $\tilde{H}$, H, and their derivatives, we get the following equations for G and H (letting $\alpha = 1$ in the previous equations):

$$\begin{aligned}
&-rG(x,y) + \pi(x) - G_x(x,y)x\Lambda(x,y) + G_y(x,y)x\Lambda(x,y) \\
&\quad + (1-q)\eta(H(x,y) - G(x,y)) - \eta G(x,y) + \eta(G(x,0) - G(x,y)) \\
&\quad + \frac{1}{2}\sigma^2[G_{xx}(x,y)x^2 + G_{yy}(x,y)y^2] + \sigma^2[xyRG_{xy}(x,y) + xG_x(x,y) + yRG_y(x,y)] = 0
\end{aligned} \tag{66}$$

and

$$
\begin{aligned}
&- rH(x,y) + \pi(y) - H_x(x,y)x\Lambda(x,y) + H_y(x,y)x\Lambda(x,y) \\
&\qquad + \eta(G(y,0) - H(x,y)) - \eta H(x,y) + \frac{1}{2}\sigma^2[H_{xx}(x,y)x^2 + H_{yy}(x,y)y^2] \\
&\qquad\qquad + \sigma^2[xyRH_{xy}(x,y) + xRH_x(x,y) + yH_y(x,y)] = 0. \quad (67)
\end{aligned}
$$

If π is homogeneous of degree $-\gamma$, then so is F. In this case, letting $f(z) = F(z,1)$, we have $F_{xx}(x,y) = y^{-\gamma-2}f''\left(\frac{x}{y}\right)$,

$$
F_{xy}(x,y) = -(\gamma+1)y^{-\gamma-2}f'\left(\frac{x}{y}\right) - xy^{-\gamma-3}f''\left(\frac{x}{y}\right),
$$

and

$$
F_{yy}(x,y) = \gamma(\gamma+1)y^{-\gamma-2}f\left(\frac{x}{y}\right) + 2(\gamma+1)xy^{-\gamma-3}f'\left(\frac{x}{y}\right) + x^2y^{-\gamma-4}f''\left(\frac{x}{y}\right).
$$

This implies that, at $(x,y) = (z,1)$,

$$
\begin{aligned}
&\frac{1}{2}\sigma^2[F_{xx}(x,y)x^2 + F_{yy}(x,y)y^2 + 2zF_{xy}(x,y)] \\
&\qquad = \sigma^2\left[\frac{\gamma}{2}(\gamma+1)f(z) + (\gamma+1)(1-R)f'(z) + (1-R)z^2f''(z)\right]. \quad (68)
\end{aligned}
$$

With $\gamma = 1$, this reduces at $(x,y) = (z,1)$ to

$$
\begin{aligned}
&\frac{1}{2}\sigma^2[F_{xx}(x,y)x^2 + F_{yy}(x,y)y^2 + 2xyF_{xy}(x,y)] \\
&\qquad = \sigma^2[f(z) + 2(1-R)zf'(z) + (1-\rho)z^2f''(z)]. \quad (69)
\end{aligned}
$$