A Derivation of $\pi_{\text{min}}(\overline{\pi})$

Recall the expression for the fraction of wrongful convictions under DPP, which equals $\overline{\pi}$ under the proposed $\pi^{**}$, we have

$$
\overline{\pi} = \frac{(1 - \pi^{**})^2 R}{(1 - \pi^{**})R + \pi^{**}},
$$

(A.1)

which solves

$$
\pi^{**} = \frac{2R\overline{l} + R + 1 - \sqrt{(R + 1)^2 + 4R\overline{l}}}{2R(\overline{l} + 1)}.
$$

(A.2)

where $\overline{l} \equiv \frac{1 - \pi}{\pi}$. The remainder of the proof computes the expected number of crimes, which is

$$
\frac{2\pi^{**}}{(1 - \pi^{**})R + \pi^{**}} = \frac{(1 - \pi^{**})^2 R}{(1 - \pi^{**})R + \pi^{**}} \cdot \frac{2\pi^{**}}{(1 - \pi^{**})^2 R} = (1 - \pi) \frac{2\pi^{**}}{(1 - \pi^{**})^2 R}.
$$

(A.3)

Plugging in (A.2), we have

$$(1 - \pi^{**})^2 R = \frac{\left(R - 1 + \sqrt{(R + 1)^2 + 4R\overline{l}}\right)^2}{4R(\overline{l} + 1)^2} = \frac{\left(R - 1 + \sqrt{(R + 1)^2 + 4R\overline{l}}\right)^2 (R - 1 - \sqrt{(R + 1)^2 + 4R\overline{l}})^2}{4R(\overline{l} + 1)^2 (R - 1 - \sqrt{(R + 1)^2 + 4R\overline{l}})^2}.$$

$$
= \frac{(R - 1)^2 - (R + 1)^2 - 4R\overline{l}}{4R(\overline{l} + 1)^2 (R - 1 - \sqrt{(R + 1)^2 + 4R\overline{l}})^2} = \frac{16R^2(\overline{l} + 1)^2}{4R(\overline{l} + 1)^2 (R - 1 - \sqrt{(R + 1)^2 + 4R\overline{l}})^2},
$$

(A.4)

as well as

$$
2\pi^{**} = \frac{2R\overline{l} + R + 1 - \sqrt{(R + 1)^2 + 4R\overline{l}}}{R(\overline{l} + 1)}.
$$
Therefore,

\[
\frac{2\pi^{**}}{(1 - \pi^{**})^2 R} = \frac{1}{4R^2(l + 1)} \left(2Rl + R + 1 - \sqrt{(R + 1)^2 + 4Rl} \right) \left(R - 1 - \sqrt{(R + 1)^2 + 4Rl} \right)^2.
\]

\[
= \frac{1}{4R^2(l + 1)} \left(2Rl + R + 1 - \sqrt{(R + 1)^2 + 4Rl} \right) \left((R - 1)^2 + (R + 1)^2 + 4Rl - 2(R - 1)\sqrt{(R + 1)^2 + 4Rl} \right)
\]

\[
= \frac{1}{2R^2(l + 1)} \left(2Rl + R + 1 - \sqrt{(R + 1)^2 + 4Rl} \right) \left(2Rl + R^2 + 1 - (R - 1)\sqrt{(R + 1)^2 + 4Rl} \right)
\]

\[
= \frac{1}{2R^2(l + 1)} \left((2Rl+R+1)(2Rl+R^2+1)+(R-1)((R+1)^2+4Rl)-(R-1)(2Rl+R+1)+2Rl+R^2+1\right)\sqrt{(R + 1)^2 + 4Rl}
\]

\[
= 2l + R + 1 - \sqrt{(R + 1)^2 + 4Rl}
\]

Since \(\pi = \frac{1}{1+l}\), we have

\[
\frac{2\pi^{**}}{(1 - \pi^{**})R + \pi^{**}} = \frac{2l + R + 1 - \sqrt{(R + 1)^2 + 4Rl}}{1 + l}.
\]