A Technology-Gap Model of Premature Deindustrialization

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Introduction
Structural Change

As per capita income rises, employment or value-added shares

- fall in Agriculture
- rise in Services
- rise and fall in Manufacturing

From Herrendorf-Rogerson-Valentinyi (2014)

Evidence from Long Time Series for the Currently Rich Countries
(Belgium, Finland, France, Japan, Korea, Netherlands, Spain, Sweden, United Kingdom, and United States) 1800-2000
Premature Deindustrialization (Rodrik, J. Econ Growth, 2016)

Late (recent) industrializers start deindustrializing at lower per capita income levels with the lower peak manufacturing sector shares, compared to richer early industrializers.

Fig. 5  Income at which manufacturing employment peaks (logs)
This Paper: A simple model of Premature Deindustrialization.

3 Goods/Sectors, 1 = Agriculture, 2 = Manufacturing, 3 = Services, homothetic CES with gross complements.

Frontier Technology: productivity growing at $g_1 > g_2 > g_3 > 0$, causing a decline of agriculture, a rise of services, and a hump-shaped path of manufacturing in each country, as in Ngai-Pissarides (2007)

Adoption Lags: $(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$

$\lambda \geq 0$: Technology Gap: country-specific, as in Krugman (1985)

$\theta_j > 0$: sector-specific, common across countries

- Countries differ in one dimension, $\lambda$, in their ability to adopt the frontier technologies.
- How the technology gap affects its adoption lags might vary across sectors, $\theta_j > 0$,

We show, for example,

- No premature deindustrialization if $\theta_1 = \theta_2 = \theta_3$
- Premature deindustrialization if $\theta_1 = \theta_2 < \theta_3 < g_2\theta_2/g_3 < g_1\theta_1/g_3$

i) Technology adoption takes longer in services

ii) Despite the longer adoption lag in services, its productivity growth is sufficiently smaller that cross-country productivity differences smaller in services: e.g., Kravis-Heston-Summers (1982).

Extensions with a catching-up (an exponential decay in $\lambda$) or nonhomothetic CES do not affect the messages.
The Model
Three Complementary Goods/Competitive Sectors, $j = 1, 2, 3$

Sector-1 = Agriculture, Sector-2 = Manufacturing, Sector-3 = Services.

Demand System:

$L$ Identical Households, each supplying one unit of mobile labor at $w$; $\kappa_j$ units of factor specific to sector-$j$ at $\rho_j$.

Budget Constraint:

$$\sum_{j=1}^{3} p_j c_j \leq E \equiv w + \sum_{j=1}^{3} \rho_j \kappa_j$$

CES Preferences:

$$U(c) = \left[ \sum_{j=1}^{3} (\beta_j)^{1/\sigma} (c_j)^{1-1/\sigma} \right]^{\sigma-1}$$

with $\beta_j > 0$ and $0 < \sigma < 1$ (gross complementarity)

Expenditure Shares:

$$m_j = \frac{p_j c_j}{E} = \frac{\beta_j (p_j)^{1-\sigma}}{\sum_{k=1}^{3} \beta_k (p_k)^{1-\sigma}} = \beta_j \left( \frac{E/p_j}{U} \right)^{\sigma-1}$$
Three Competitive Sectors: Production

Cobb-Douglas

\[ Y_j = A_j \left( \kappa_j L_j \right)^\alpha \left( L_j \right)^{1-\alpha} \]

\( A_j > 0 \): the TFP of sector-\( j \); \( \alpha \in [0,1) \) the share of specific factor.

Output per worker

\[ \frac{Y_j}{L_j} = \bar{A}_j (s_j)^{-\alpha} \]

Output per capita

\[ \frac{Y_j}{L} = \bar{A}_j (s_j)^{1-\alpha} \]

where \( \bar{A}_j \equiv A_j \left( \kappa_j \right)^\alpha \) and

Employment Shares

\[ s_j \equiv \frac{L_j}{L}; \quad \sum_{j=1}^{3} s_j = 1 \]

With Cobb-Douglas, \( wL_j = (1 - \alpha)p_j Y_j \), implying the employment shares equal to

Value-Added Shares

\[ \frac{p_j Y_j}{EL} = \frac{p_j Y_j}{\sum_{k=1}^{3} p_k Y_k} = s_j = \frac{L_j}{L} \]
**Equilibrium**  The expenditure shares are equal to the value-added shares.

\[ m_j = \frac{p_j Y_j}{E L} = s_j \]

which lead to

**Equilibrium Shares**

\[ s_j = \frac{\left[ \beta_j \frac{1}{\sigma - 1} \bar{A}_j \right]^{-a}}{\sum_{k=1}^{3} \left[ \beta_k \frac{1}{\sigma - 1} \bar{A}_k \right]^{-a}} \]

**Per Capita Income**

\[ U = \left\{ \sum_{k=1}^{3} \left[ \beta_k \frac{1}{\sigma - 1} \bar{A}_k \right]^{-a} \right\}^{-\frac{1}{a}} \]

where

\[ a \equiv \frac{1 - \sigma}{1 - \alpha(1 - \sigma)} > 0 \]
Productivity Growth Rates, Adoption Lags and Structural Change
Suppose that \( \{\tilde{A}_j(t)\}_{j=1}^3 \) change according to:

\[
\tilde{A}_j(t) = \tilde{A}_j(t - \lambda_j) = \tilde{A}_j(0)e^{g_j(t-\lambda_j)} = \tilde{A}_j(0)e^{-\lambda_j g_j e^{g_j t}}
\]

\( \tilde{A}_j(t) = \tilde{A}_j(0)e^{g_j t} \): Frontier Technology in \( j \), with a constant growth rate \( g_j > 0 \).

\( \tilde{A}_j(t) = \tilde{A}_j(t - \lambda_j) \); \( \lambda_j = \text{Adoption Lag} \) in \( j \).

• \( g_j \) and \( \lambda_j \) are sector-specific.

• \( \lambda_j \) has no “growth” effect.

• \( \lambda_j \) has the “level” effect, \( e^{-\lambda_j g_j} \), which depends on \( g_j \).

A large adoption lag would not matter much in a sector with slow productivity growth. Even a small adoption lag would matter a lot in a sector with fast productivity growth.
\[ U(t) = \left\{ \sum_{k=1}^{3} \tilde{\beta}_k e^{-ag_k(t-\lambda_k)} \right\}^{-\frac{1}{a}} \]

where \( \tilde{\beta}_k \equiv \left( \beta_k^{\frac{1}{\sigma-1}} \bar{A}_k(0) \right)^{-a} > 0 \). Longer adoption lags would shift down the time path of \( U(t) \).

\[ \frac{s_j(t)}{s_k(t)} = \left( \frac{\beta_j^{\frac{1}{\sigma-1}} \bar{A}_j(t - \lambda_j)}{\beta_k^{\frac{1}{\sigma-1}} \bar{A}_k(t - \lambda_k)} \right)^{-a} = \frac{\tilde{\beta}_j}{\tilde{\beta}_k} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \]

**Relative Growth Effect:** \( s_j(t)/s_k(t) \) is decreasing over time if \( g_j > (<) g_k \).

**Shift from faster growing sectors to slower growing sectors over time.**

**Relative Level Effect:** A higher \( \lambda_j g_j - \lambda_k g_k \) raises \( s_j(t)/s_k(t) \) at any point in time.

\[ \left( \frac{p_j(t)}{p_k(t)} \right)^{1-\alpha(1-\sigma)} = \left( \frac{\beta_j}{\beta_k} \right)^{\alpha} \frac{\bar{A}_k(0)}{\bar{A}_j(0)} e^{(\lambda_j g_j - \lambda_k g_k)} e^{(g_k - g_j)t} \]

**Relative Growth Effect:** \( p_j(t)/p_k(t) \) is decreasing over time if \( g_j > (<) g_k \).

**Relative Level Effect:** A higher \( \lambda_j g_j - \lambda_k g_k \) raises \( p_j(t)/p_k(t) \) at any point in time.
Structural Change a la Ngai-Pissarides (2007): Let $g_1 > g_2 > g_3 > 0$

Decline of Agriculture: $s_1(t)$ is decreasing over time, because

$$\frac{1}{s_1(t)} = \frac{s_1(t) + s_2(t) + s_3(t)}{s_1(t)} = 1 + \frac{s_2(t)}{s_1(t)} + \frac{s_3(t)}{s_1(t)}$$

Rise of Services: $s_3(t)$ is increasing over time, because

$$\frac{1}{s_3(t)} = \frac{s_1(t) + s_2(t) + s_3(t)}{s_3(t)} = \frac{s_1(t)}{s_3(t)} + \frac{s_2(t)}{s_3(t)} + 1$$

Rise and Fall of Manufacturing: $s_2(t)$ is hump-shaped, because

$$\frac{1}{s_2(t)} = \frac{s_1(t) + s_2(t) + s_3(t)}{s_2(t)} = \frac{s_1(t)}{s_2(t)} + 1 + \frac{s_3(t)}{s_2(t)}$$

$$= \left[ \frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\lambda_1 g_1 - \lambda_2 g_2)} \right] e^{-a(g_1 - g_2)t} + 1 + \left[ \frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\lambda_3 g_3 - \lambda_2 g_2)} \right] e^{a(g_2 - g_3)t},$$

where $\tilde{\beta}_j \equiv \left( \frac{1}{\sigma - 1} \tilde{A}_j(0) \right)^{-a} > 0$. 

Manufacturing Peak: “^” indicates the peak. From \( s'_2(\hat{t}) = 0 \),

\[
\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} + \hat{t}_0, \quad \text{where} \quad \left( \frac{\beta_1}{\beta_3} \right)^{\frac{1}{\sigma-1}} \frac{\bar{A}_1(\hat{t}_0)}{\bar{A}_3(\hat{t}_0)} \equiv \left( \frac{g_1 - g_2}{g_2 - g_3} \right)^{\frac{1}{\bar{a}}}
\]

We reset the calendar time such that \( \hat{t}_0 = 0 \). This is equivalent to the following normalization:

\[
\left( \frac{g_1 - g_2}{g_2 - g_3} \right) \frac{\tilde{\beta}_1}{\tilde{\beta}_3} = \left( \frac{g_1 - g_2}{g_2 - g_3} \right) \left( \frac{\beta_1}{\beta_3} \right)^{\frac{1}{\sigma-1}} \frac{\bar{A}_1(0)}{\bar{A}_3(0)} = 1.
\]

Then,

**Peak Time**

\[
\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3}.
\]

**Peak M-Share**

\[
\frac{1}{s_2(\hat{t})} = 1 + \left( \frac{\tilde{\beta}_1 + \tilde{\beta}_3}{\tilde{\beta}_2} \right) e^{\frac{a(\lambda_1 - \lambda_2) g_1 g_2 + (\lambda_2 - \lambda_3) g_2 g_3 + (\lambda_3 - \lambda_1) g_3 g_1}{g_1 - g_3}}
\]

**Peak Time Per Capita Income**

\[
U(\hat{t}) = \left\{ \left( \tilde{\beta}_1 + \tilde{\beta}_3 \right) e^{-a\frac{(\lambda_1 - \lambda_3) g_1 g_3}{g_1 - g_3}} + \tilde{\beta}_2 e^{-a\frac{(\lambda_1 - \lambda_2) g_1 g_2 + (\lambda_2 - \lambda_3) g_2 g_3}{g_1 - g_3}} \right\}^{\frac{1}{\bar{a}}}
\]
Technology Gaps and Premature Deindustrialization
Many countries with

\[(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3) \lambda\]

\(\lambda \geq 0: \text{Technology Gap: Country-specific.}\)

\(\theta_j > 0: \text{Sector-specific, common across countries}\)

- Countries differ in one dimension, \(\lambda\), in their ability to adopt the frontier technologies.
- How the technology gap affects its adoption lags might vary across sectors, \(\theta_j > 0\), which captures the inherent difficulty of adoption in each sector.
**Benchmark Case:** Uniform Adoption Lags, as in Krugman (1985)

\[
\begin{align*}
\theta_1 = \theta_2 = \theta_3 &= 1 \iff \lambda_1 = \lambda_2 = \lambda_3 = \lambda > 0 \\
\Rightarrow \hat{t} &= \lambda; \quad \frac{1}{s_2(\hat{t})} = 1 + \left(\frac{\tilde{\beta}_1 + \tilde{\beta}_3}{\tilde{\beta}_2}\right); \quad U(\hat{t}) = \left(\tilde{\beta}_1 + \tilde{\beta}_2 + \tilde{\beta}_3\right)^{-1/a}
\end{align*}
\]

The country’s technology gap causes a delay in the peak time, \( \hat{t} \), by \( \lambda > 0 \).
The peak manufacturing share & per capita income at the peak time unaffected.

Each country follows the same development path of early industrializers with a delay.

No Premature Deindustrialization!!
Premature Deindustrialization: An Example

\[ \theta_1 = \theta_2 = \theta < \theta_3 = 1 \implies \lambda_1 = \lambda_2 < \lambda_3 \]

Technology adoption harder in services (due to its intangible nature of technology).

\[ g_3 / g_2 < \theta < 1 \implies \lambda_1 g_1, \lambda_2 g_2 > \lambda_3 g_3 \]

Cross-country productivity differences smaller in services: e.g., Kravis-Heston-Summers (1982).

\[ \hat{t} = \left( \frac{\theta g_1 - g_3}{g_1 - g_3} \right) \lambda \]

\[ \frac{1}{s_2(\hat{t})} = 1 + \left( \frac{\tilde{\beta}_1 + \tilde{\beta}_3}{\tilde{\beta}_2} \right) e^{\frac{a(g_1 - g_2)g_3}{g_1 - g_3}(1-\theta)\lambda} \]

\[ U(\hat{t}) = \left\{ \left( \tilde{\beta}_1 + \tilde{\beta}_3 \right) e^{\frac{a g_1 g_3}{g_1 - g_3}(1-\theta)\lambda} + \tilde{\beta}_2 e^{\frac{a g_2 g_3}{g_1 - g_3}(1-\theta)\lambda} \right\}^{-\frac{1}{a}} \]

\[ \frac{\partial \hat{t}}{\partial \lambda} > 0 \]

\[ \frac{\partial s_2(\hat{t})}{\partial \lambda} < 0 \]

\[ \frac{\partial U(\hat{t})}{\partial \lambda} < 0 \]
Figure 1: Premature Deindustrialization: A higher $\lambda \rightarrow$ peak later at a lower share and at a lower income.

Ten trajectories plotted for $\lambda = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$, with $g_1 = .10 > g_2 = .06 > g_3 = .02$, $\theta = .35$, $\alpha = 1/3$, and $\sigma = 0.6$ (hence $a = 6/13$).

$\beta_j = 1$, $\bar{A}_j(0) = 3^{1/a} = 3^{13/6} = 10.8...$, so that $\tilde{\beta}_j = 1/3$ for $j = 1,2,3$, hence $\tilde{U}(\hat{t}) = 1$; $\hat{t} = 0$ for $\lambda = 0$. 
Conditions for Premature Deindustrialization

\( \hat{t} \) increasing in \( \lambda \) if

\[ \frac{\theta_1}{\theta_3} > \frac{g_3}{g_1}; \]

\( \hat{s}_2 = s_2(\hat{t}) \) decreasing in \( \lambda \) if:

\[ (1 - \frac{g_3}{g_2})\left(\frac{\theta_1}{\theta_3} - \frac{g_3}{g_1}\right) > (1 - \frac{g_3}{g_1})\left(\frac{\theta_2}{\theta_3} - \frac{g_3}{g_2}\right); \]

\( \hat{U} = U(\hat{t}) \) decreasing in \( \lambda \) if:

\[ (1 - \frac{g_3}{g_1})\frac{\theta_2}{\theta_3} > (\frac{\theta_1}{\theta_3} - \frac{g_3}{g_1}); \]
Two Extensions
Narrowing a Technology Gap

We assumed that $\lambda$ is time-invariant. This implies

The sectoral productivity growth rate is constant over time & identical across countries.
[In contrast, the aggregate growth rate, $g_U(t) \equiv U'(t)/U(t) = \sum_{k=1}^{3} g_k s_k(t)$, declines over time, $g'_U(t) = g_1 s_1'(t) + g_2 s_2'(t) + g_3 s_3'(t) = (g_1 - g_2)s_1'(t) + (g_3 - g_2)s_3'(t) < 0$, the so-called Baumol’s cost disease.]

What if latecomers can narrow a technology gap, and hence achieve a higher productivity growth in each sector?

Let countries differ in the initial value of lambda, $\lambda_0$, but they converge exponentially over time at the same rate,

$$\bar{A}_j(t) = \bar{A}_j(0) e^{g_j(t - \theta_j \lambda_t)}, \quad \text{where} \quad \lambda_t = \lambda_0 e^{-g_{\lambda} t}, \quad g_{\lambda} > 0.$$
Again, by setting the calendar time such that \( \hat{t}_0 = 0 \) for the frontier country with \( \lambda_0 = 0 \),

**Peak Time**

\[
\hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda \hat{t} + D(g \lambda \hat{t})
\]

**Peak Share**

\[
\frac{1}{s_2(\hat{t})} = 1 + \left( \frac{\tilde{\beta}_1 + \tilde{\beta}_3}{\tilde{\beta}_2} \right) e^{a[(\theta_1 - \theta_2) g_1 g_2 + (\theta_2 - \theta_3) g_2 g_3 + (\theta_3 - \theta_1) g_3 g_1] \lambda \hat{t}} \left[ \frac{(g_2 - g_3) e^{a(g_2 - g_1) D(g \lambda \hat{t})} + (g_1 - g_2) e^{a(g_2 - g_3) D(g \lambda \hat{t})}}{g_1 - g_3} \right]
\]

**Peak Time Per Capita Income**

\[
U(\hat{t}) = \left\{ \left( \tilde{\beta}_1 e^{-a g_1 D(g \lambda \hat{t})} + \tilde{\beta}_3 e^{-a g_3 D(g \lambda \hat{t})} \right) e^{-a(\theta_1 - \theta_3) g_1 g_3 \lambda \hat{t}} + \left( \tilde{\beta}_2 e^{-a g_2 D(g \lambda \hat{t})} \right) e^{-a(\theta_1 - \theta_2) g_1 g_2 + (\theta_2 - \theta_3) g_2 g_3 \lambda \hat{t}} \right\}^{-\frac{1}{a}}
\]

where

\[
D(g \lambda \hat{t}) = \frac{1}{a(g_1 - g_3)} \ln \left[ \frac{g_1 - g_2 + (\theta_1 g_1 - \theta_2 g_2) g \lambda \hat{t}}{g_2 - g_3 - (\theta_3 g_3 - \theta_2 g_2) g \lambda \hat{t}} \right] \frac{(g_2 - g_3)}{g_1 - g_2}.
\]

For \( g_\lambda = 0, D(g \lambda \hat{t}) = D(0) = 0 \), and all the parts in red disappear, and we go back to the baseline model.
**Peak Time**: Latecomers peak later.

**Peak Share**: Latecomers peak at lower shares.

**Peak time Income**: Latecomers peak at lower income for a realistic range of $g_\lambda (< 4\%)$: Comin-Mestieri (2018)
**Isoelastic Nonhomothetic CES;** Comin-Lashkari-Mestieri (2021), Matsuyama (2019)

\[
\left[ \sum_{j=1}^{3} \left( \beta_j \right)^{1/\sigma} \left( \frac{c_j}{U^{\varepsilon_j}} \right)^{1-1/\sigma} \right]^{\sigma-1} \equiv 1
\]

Normalize \( \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3 \); with \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1 \), we go back to the case of homothetic CES.

With \( \sigma < 1 \), \( 0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 \) would imply that agriculture (service) has the lowest (highest) income elasticity.

By maximizing \( U \) subject to \( \sum_{j=1}^{3} p_j c_j \leq E \),

**Expenditure Shares**

\[
m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j \left( \frac{U^{\varepsilon_j} p_j}{E} \right)^{1-\sigma}}{\sum_{k=1}^{3} \beta_k \left( \frac{U^{\varepsilon_k} p_k}{E} \right)^{1-\sigma}} = \beta_j \left( \frac{U^{\varepsilon_j} p_j}{E} \right)^{1-\sigma}
\]

**Indirect Utility Function:**

\[
\left[ \sum_{j=1}^{3} \beta_j \left( \frac{U^{\varepsilon_j} p_j}{E} \right)^{1-\sigma} \right]^{1/1-\sigma} \equiv 1
\]

**Cost-of-Living Index:**

\[
\left[ \sum_{j=1}^{3} \beta_j \left( \frac{U^{\varepsilon_j} p_j}{PU} \right)^{1-\sigma} \right]^{1/1-\sigma} \equiv 1 \iff U \equiv \frac{E}{P}
\]

**Income Elasticity:**

\[
\eta_j \equiv \frac{\partial \ln c_j}{\partial \ln (U)} = 1 + \frac{\partial \ln m_j}{\partial \ln (E/P)} = 1 + (1 - \sigma) \left\{ \varepsilon_j - \sum_{k=1}^{3} m_k \varepsilon_k \right\}
\]
The production side is the same as before. By following the same step, we obtain

**Equilibrium Shares**

\[ s_j = \frac{1}{\left[ U_j^\alpha \right]^{-\alpha} \left[ \beta_j \right]^{\alpha-1} A_j} \],

where \[ \sum_{k=1}^{3} \frac{1}{\left[ U_k^\alpha \right]^{-\alpha} \left[ \beta_k \right]^{\alpha-1} A_k} \equiv 1 \]

With \[ \tilde{A}_j(t) = A_j(t - \lambda_j) = A_j(0)e^{g_j(t-\theta_j \lambda)} \],

\[ s_2(t) = \frac{1}{s_2(t)} = U(t)^a(\varepsilon_1-\varepsilon_2) \left[ \begin{array}{c} \tilde{\beta}_1 \\ \tilde{\beta}_2 \\ \tilde{\beta}_3 \end{array} \right] e^{a(\theta_1 g_1 - \theta_2 g_2) \lambda} e^{-a(g_1-g_2)t} + 1 + U(t)^a(\varepsilon_3-\varepsilon_2) \left[ \begin{array}{c} \tilde{\beta}_3 \\ \tilde{\beta}_2 \\ \tilde{\beta}_1 \end{array} \right] e^{a(\theta_3 g_3 - \theta_2 g_2) \lambda} e^{-a(g_2-g_3)t} \]

\[ U(t) = U(t)^a(\varepsilon_1 \beta_1 e^{-a g_1(t-\theta_1 \lambda)} + U(t)^a(\varepsilon_2 \beta_2 e^{-a g_2(t-\theta_2 \lambda)} + U(t)^a(\varepsilon_3 \beta_3 e^{-a g_3(t-\theta_3 \lambda)}) \equiv 1 \]

\[ s_2'(t) = 0: \]

\[ g_1 - g_2 = (g_2 - g_3)U^a(\varepsilon_3-\varepsilon_2) \left[ \begin{array}{c} \tilde{\beta}_3 \\ \tilde{\beta}_2 \\ \tilde{\beta}_1 \end{array} \right] e^{a(\theta_3 g_3 - \theta_1 g_1) \lambda} e^{a(g_1-g_3)t} \]

\[ \{ (\varepsilon_1 - \varepsilon_2) + (\varepsilon_3 - \varepsilon_2)U^a(\varepsilon_3-\varepsilon_1) \left[ \begin{array}{c} \tilde{\beta}_3 \\ \tilde{\beta}_2 \\ \tilde{\beta}_1 \end{array} \right] e^{a(\theta_3 g_3 - \theta_1 g_1) \lambda} e^{a(g_1-g_3)t} \} \{ g_1 U^a(\varepsilon_1-\varepsilon_2) \tilde{\beta}_1 e^{-a g_1(t-\theta_1 \lambda)} + g_2 \tilde{\beta}_2 e^{-a g_2(t-\theta_2 \lambda)} + g_3 U^a(\varepsilon_3-\varepsilon_2) \tilde{\beta}_3 e^{-a g_3(t-\theta_3 \lambda)} \} \]

\[ \hat{\lambda} \text{ and } \hat{U} \text{ solve the equation for } U(t) \text{ and the equation for } s_2'(t) = 0, \text{ simultaneously.} \]

Then, \( \hat{s}_2 \) can be obtained by plugging \( \hat{\lambda} \) and \( \hat{U} \) into the equation for \( s_2(t) \)
Symmetric Case: $\varepsilon_1 = 1 - \varepsilon < \varepsilon_2 = 1 < \varepsilon_3 = 1 + \varepsilon$ for $0 < \varepsilon < 1$

The baseline case if $\varepsilon = 0$. Premature deindustrialization occurs for $\varepsilon > 0$.

A higher $\varepsilon$ causes late industrializers to have
- A further delay in the peak time
- A negligible effect on the peak share.
- A smaller decline in the peak time per capita income

In this case, the frontier country with $\lambda = 0$ is not affected.
Asymmetric Case-1: $\varepsilon_1 = 1 - \varepsilon < \varepsilon_2 = \varepsilon_3 = 1 + \varepsilon/2$ for $0 < \varepsilon < 1$

The baseline case if $\varepsilon = 0$. Premature deindustrialization occurs for $\varepsilon > 0$.
A higher $\varepsilon$ causes late industrializers to have
- A further delay in the peak time
- A larger decline in the peak share.
- A smaller decline in the peak time per capita income relative to the frontier country with $\lambda = 0$, which is also affected with a higher $\varepsilon$, causing
  - a higher $\hat{t}$
  - a higher $\hat{s}_2$
  - a higher $\hat{U}$. 
Asymmetric Case-2: \( \varepsilon_1 = 1 - \varepsilon / 2 = \varepsilon_2 < \varepsilon_3 = 1 + \varepsilon \) for \( 0 < \varepsilon < 1 \)

The baseline case if \( \varepsilon = 0 \). Premature deindustrialization occurs for \( \varepsilon > 0 \).

A higher \( \varepsilon \) causes late industrializers to have

- A further delay in the peak time
- A smaller decline in the peak share
- A smaller decline in the peak time real income

relative to the frontier country with \( \lambda = 0 \), which is also affected with a higher \( \varepsilon \), causing

- a lower \( \hat{t} \)
- a higher \( \hat{s}_2 \)
- a lower \( \bar{U} \).
Concluding Remarks
A simple model of Rodrik’s (2016) premature deindustrialization, based on

- **Countries heterogeneous in their technology gaps**, as in Krugman (1985).
- Sectors differ in the extent to which technology gap affects their adoption lags.

We find, for example, that premature deindustrialization occurs if

- Longer adoption lag in services
- Despite the longer adoption lag in services, its productivity growth rate is sufficiently smaller that cross-country productivity differences in levels smaller in services.

In the baseline model, we assumed

- **no catching up** (no cross-country difference in the sectoral productivity growth rate).
- **homothetic CES** preferences

In two extensions, we introduced

- **Narrowing a technology gap** to allow productivity in each sector to grow faster among late industrializers:
- **Nonhomothetic CES with income-elastic services and income-inelastic agricultures**, as CLM (2021) and found they do not affect the basic message.
Appendix: \( \varepsilon_1 = 1 - \varepsilon < \varepsilon_2 = 1 + \frac{\varepsilon}{3} < \varepsilon_3 = 1 + \frac{2\varepsilon}{3} \) for \( 0 < \varepsilon < 1 \) \( \Rightarrow \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2} = 4 \), as in CLM (2021).

An intermediate of Symmetric Case & Asymmetric Case-1.

The baseline case if \( \varepsilon = 0 \). Premature deindustrialization occurs for \( \varepsilon > 0 \).

A higher \( \varepsilon \) causes late industrializers to have
- A further delay in the peak time
- A larger decline in the peak share.
- A smaller decline in the peak time per capita income relative to the frontier country with \( \lambda = 0 \), which is also affected with a higher \( \varepsilon \), causing
  - a higher \( \hat{t} \)
  - a higher \( \hat{s}_2 \)
  - a higher \( \overline{U} \).