Dynamic models in Economics and Finance

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University of Urbino, Italy

Globalization and Coupled Chaotic Fluctuations of Innovation

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Introduction/Content

- **Two-country model of endogenous innovation fluctuations** proposed by *Matsuyama* [1], is based on *Deneckere-Judd* [2] model of innovation dynamics of a closed economy, and *Helpman-Krugman* [3] model of intra-industry trade between two structurally identical countries → a family of **2D piecewise smooth maps**;
- In autarky: **decoupled** innovation fluctuations → **the skew tent map** dynamics [4];
- **Coupled** innovation fluctuations: synchronized/asynchronized, regular/chaotic, symmetric/asymmetric dynamics [5].

[1] *K. Matsuyama, L. Gardini, I. Sushko* (2015) Globalization and Synchronization of Innovation Cycles (WP), http://faculty.wcas.northwestern.edu/~kmatsu/ (presented at MDEF2014).
[2] *R. Deneckere, K. Judd* (1992) Cyclical and Chaotic Behavior in a Dynamic Equilibrium Model.
[3] *E. Helpman, P. Krugman* (1985) Market Structure and International Trade.
[4] *I. Sushko, V. Avrutin, L. Gardini* (2015) Bifurcation structure in the skew tent map and its application as a border collision normal form.
[5] *C. Mira, L. Gardini, A. Barugola and J.C. Cathala* (1996) Chaotic Dynamics in Two-Dimensional Nonivertible Maps.

Definition of the map

• The two-country model of endogenous innovation fluctuations is described by a family of **2D piecewise smooth maps** $F : \mathbb{R}_+^2 \to \mathbb{R}_+^2$, $(x, y) \mapsto F(x, y)$, where x and y are **normalized measures of innovation cost per variety** in country 1 and 2, resp.;

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- These regions are separated by **four boundaries**: **(1)** $x = s_1(\rho)$, **(2)** $y = s_2(\rho)$, where $s_1(\rho) = 1 - s_2(\rho) = \min\left\{\frac{s_1-\rho s_2}{1-\rho}, 1\right\}$; **(3)** $x = h_1(y)$ and **(4)** $y = h_2(x)$ obtained from complementarity slackness conditions $\frac{s_1}{x+\rho y} + \frac{\rho s_2}{y+\rho x} = 1$ and $\frac{s_2}{y+\rho x} + \frac{\rho s_1}{x+\rho y} = 1$, resp.

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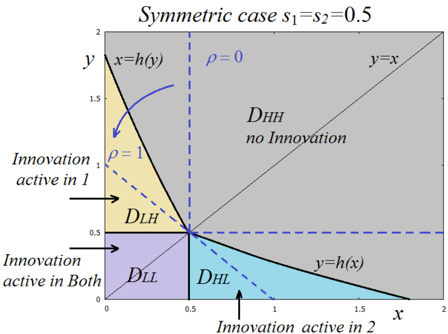

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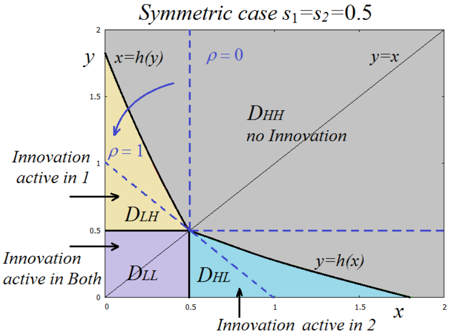

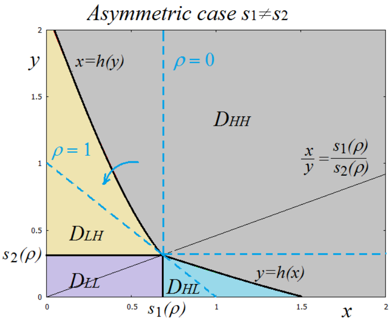

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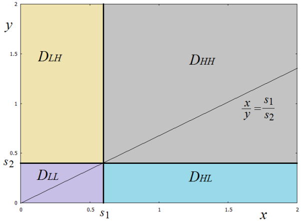
Parameters:

- $\theta \in (1, e)$ is the market share of a competitive variety relative to a monopolistic variety;
- $\rho \in (0, 1)$ is a degree of globalization;
- $\delta \in (0, 1)$ is the survival rate of old varieties;
- $s_1 = 1 - s_2 \in [0.5, 1)$, $s_1 = \frac{L_1}{L_1+L_2}$ and $s_2 = \frac{L_2}{L_1+L_2}$, where L_1 and L_2 are associated with labor supply in country 1 and 2, resp.

In autarky: Decoupled system ($\rho = 0$)



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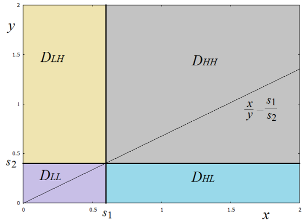

Innovation fluctuations in each country are described by the related skew tent map f:

$$x \xrightarrow{f} \begin{cases} f_L(x) = \delta(\theta s_1 + (1-\theta)x), & x \leq s_1, \\ f_H(x) = \delta x, & x > s_1, \end{cases}$$

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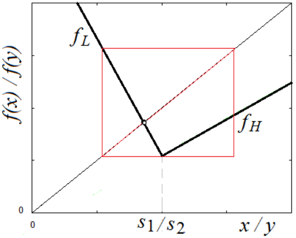

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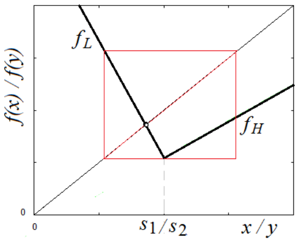

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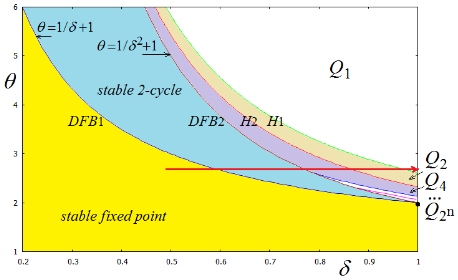

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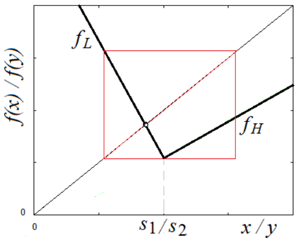

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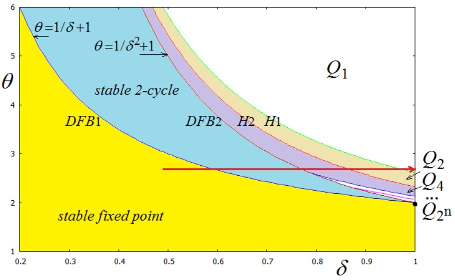

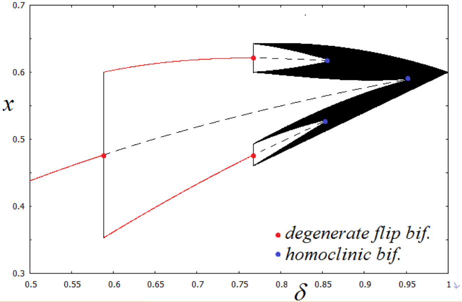

Decoupled system: a 2D view

An n-cyclic attractor (an attracting n-cycle or n-piece chaotic attractor) of the skew tent map $f \Rightarrow n$ coexisting n-cyclic attractors of F

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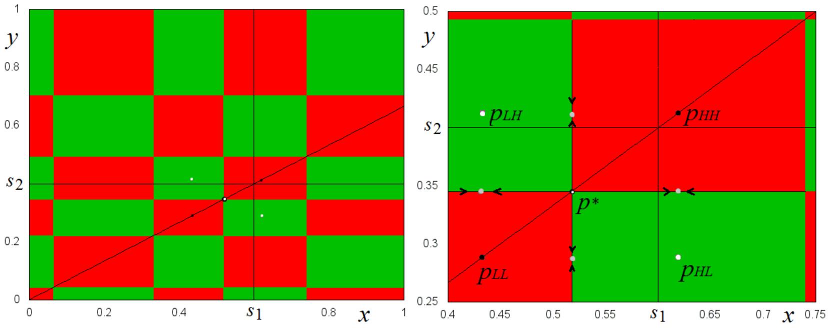
$\theta = 2.7$, $\delta = 0.7$: 2 coexisting 2-cycles



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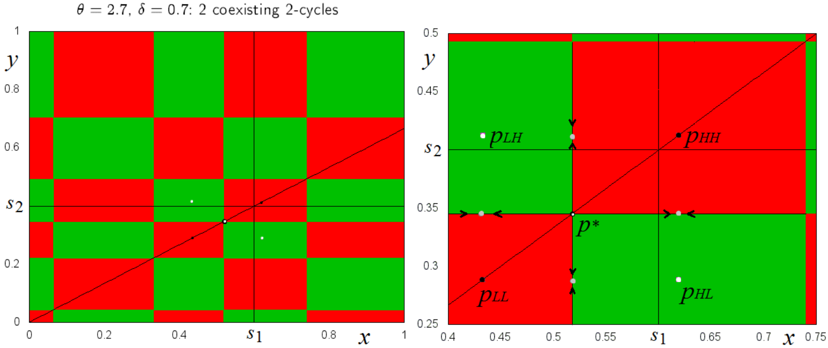
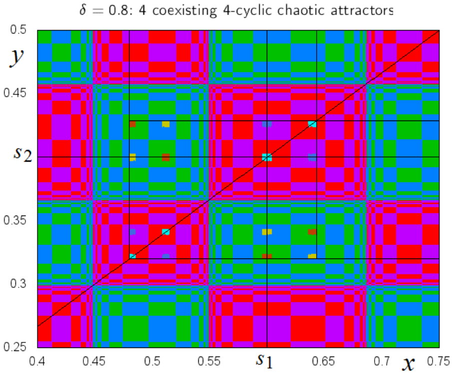


Synchronized attracting 2-cycle $\{p_{LL}, p_{HH}\}$; **Asynchronized** attracting 2-cycle $\{p_{LH}, p_{HL}\}$; their basins are separated by the closure of the stable invariant sets of two saddle 2-cycles; unique fixed point $p*$ is unstable.

Decoupled system: a 2D view (2)

An n-cyclic attractor of $f \Rightarrow n$ coexisting n-cyclic attractors of F



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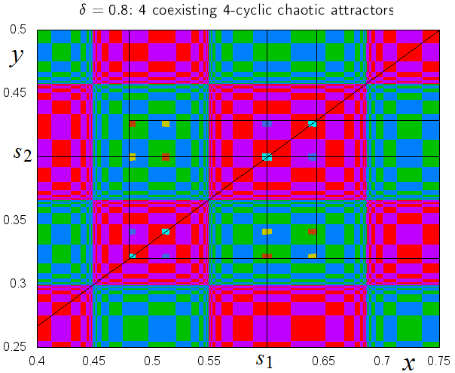
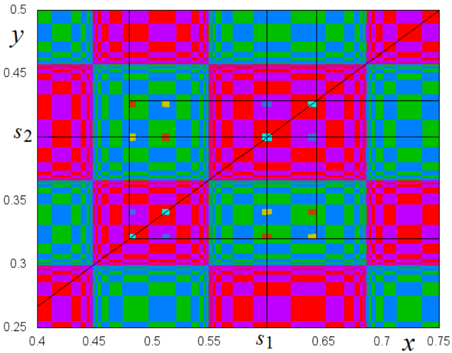


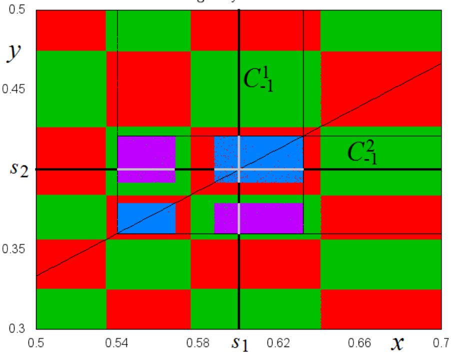
Decoupled system: a 2D view (2)

An n-cyclic attractor of $f \Rightarrow n$ coexisting n-cyclic attractors of F

$\delta = 0.8$: 4 coexisting 4-cyclic chaotic attractors



$\delta = 0.9$: 2 coexisting 2-cyclic chaotic attractors



Boundaries of the chaotic attractors are formed by the proper images of the generating segments of the **critical lines**
$C^1_{-1} : \{(x, y) : x = s_1, y > 0\}$, $C^2_{-1} = \{(x, y) : x > 0, y = s_2\}$.

Coupled system: critical lines and absorbing region

F is a **noninvertible map** of $Z_0 - Z_2 - Z_4$ type; the regions with different number of preimages are separated by the **critical lines** $C^i = F(C^i_{-1})$, $i = \overline{1,4}$, where
$C^1_{-1} : \{(x,y) : x = s_1, 0 < y < s_2(\rho)\}$, $\quad C^2_{-1} = \{(x,y) : 0 < x < s_1(\rho), y = s_2\}$,
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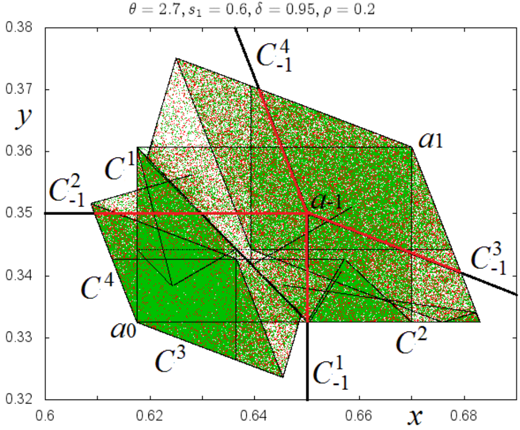
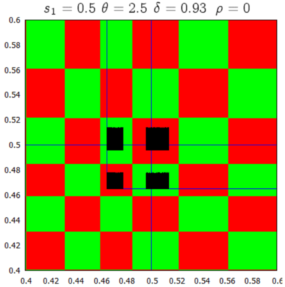


Coupled system: increasing ρ in symmetric case



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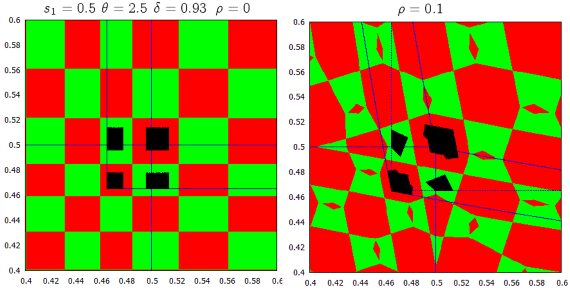

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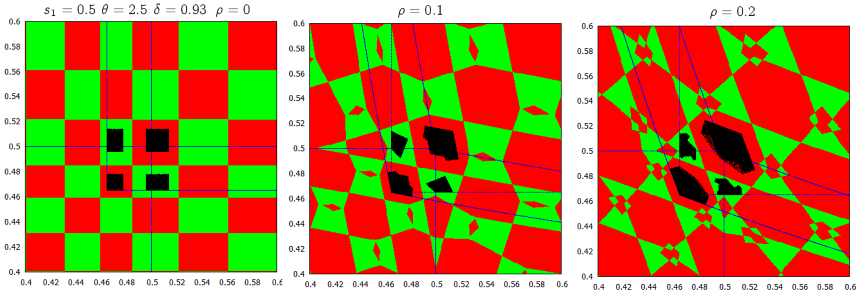

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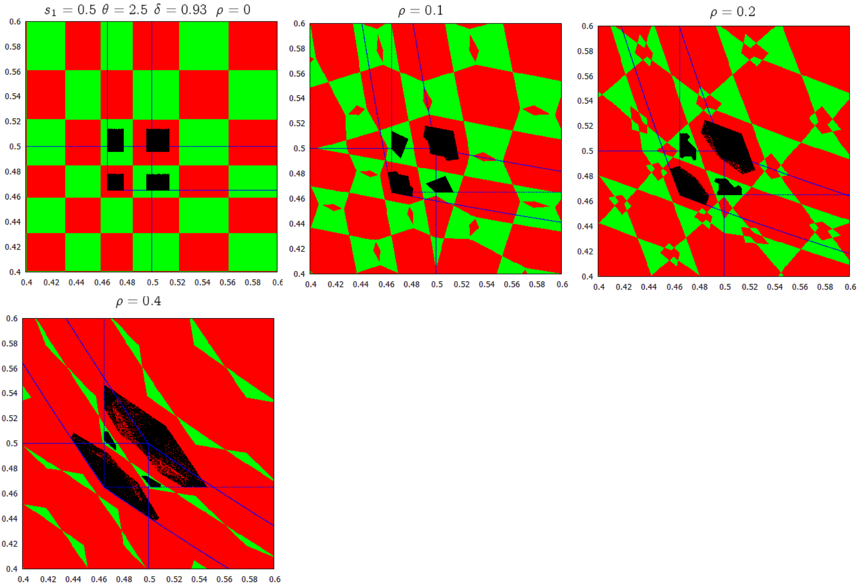

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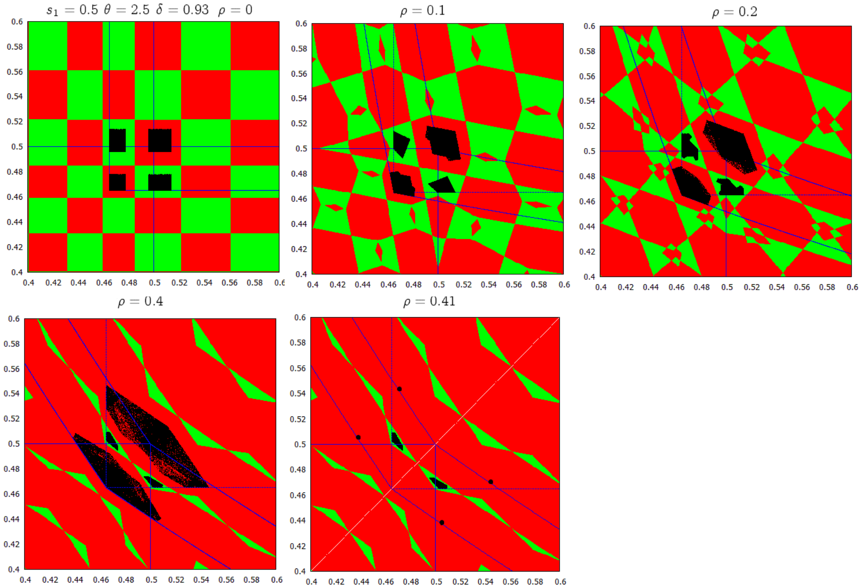

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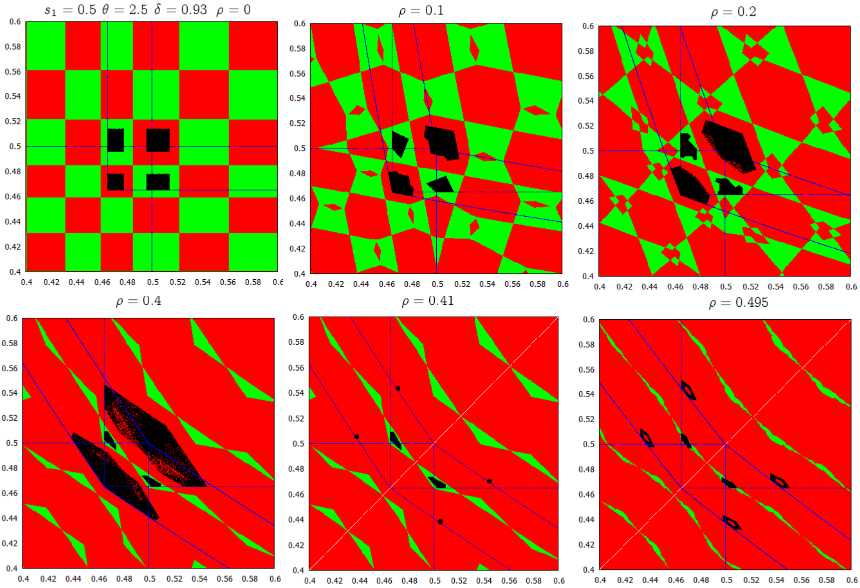
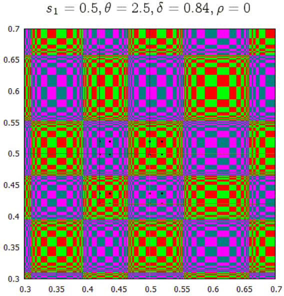


Coupled system: symmetric case (2)



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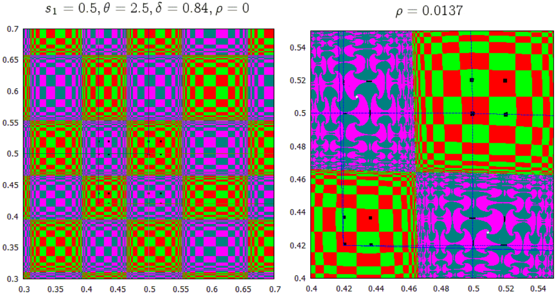
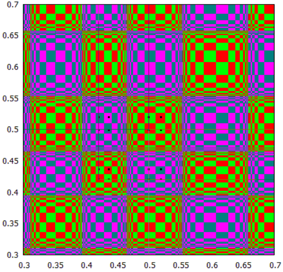
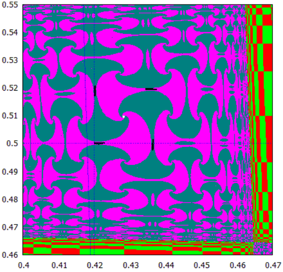


Coupled system: symmetric case (2)

$s_1 = 0.5, \theta = 2.5, \delta = 0.84, \rho = 0$



$\rho = 0.0137$



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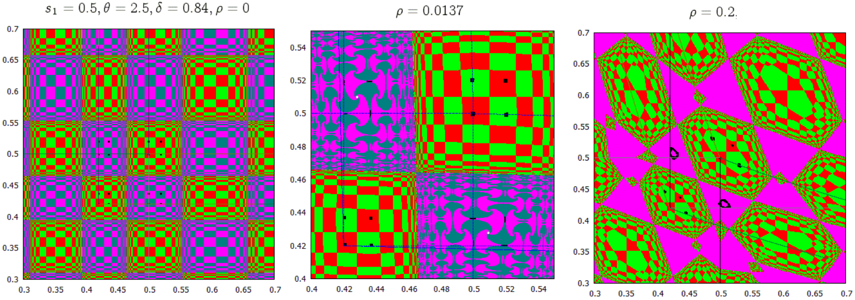

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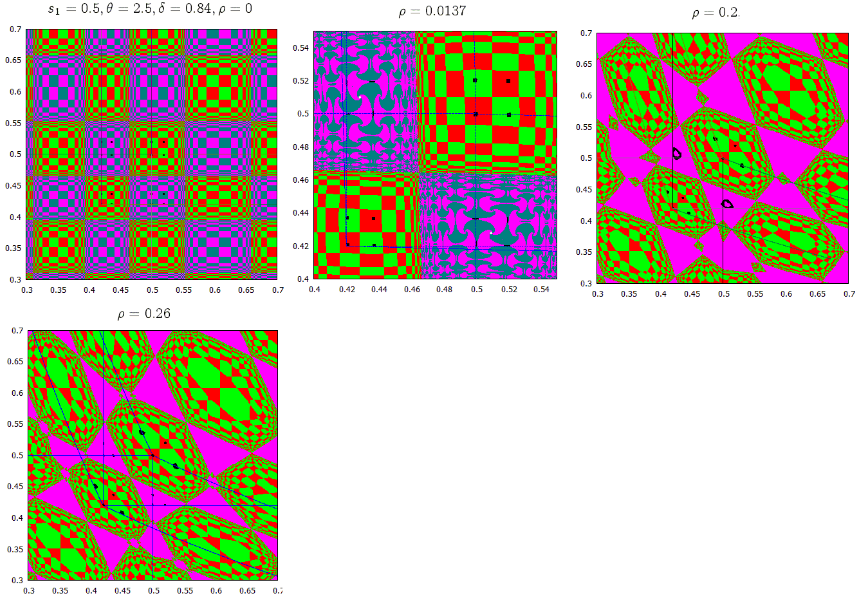

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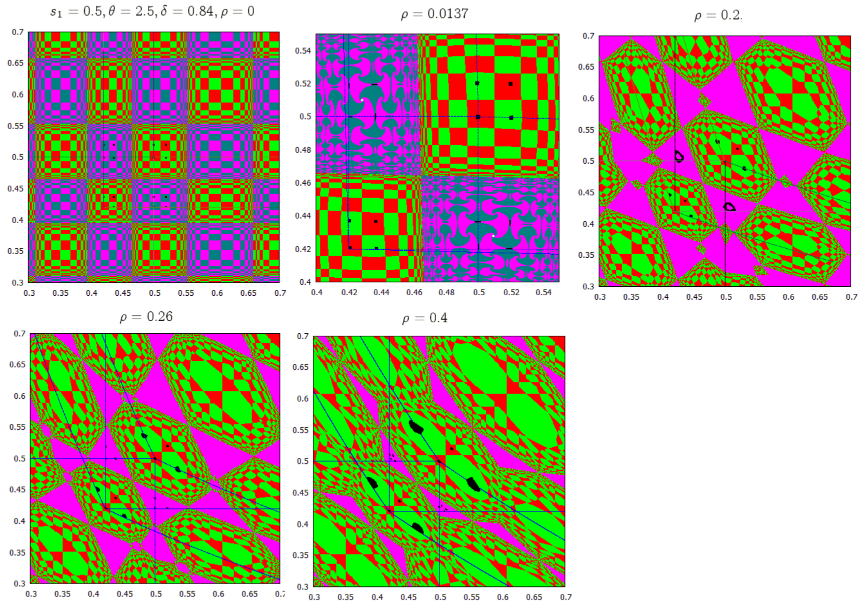
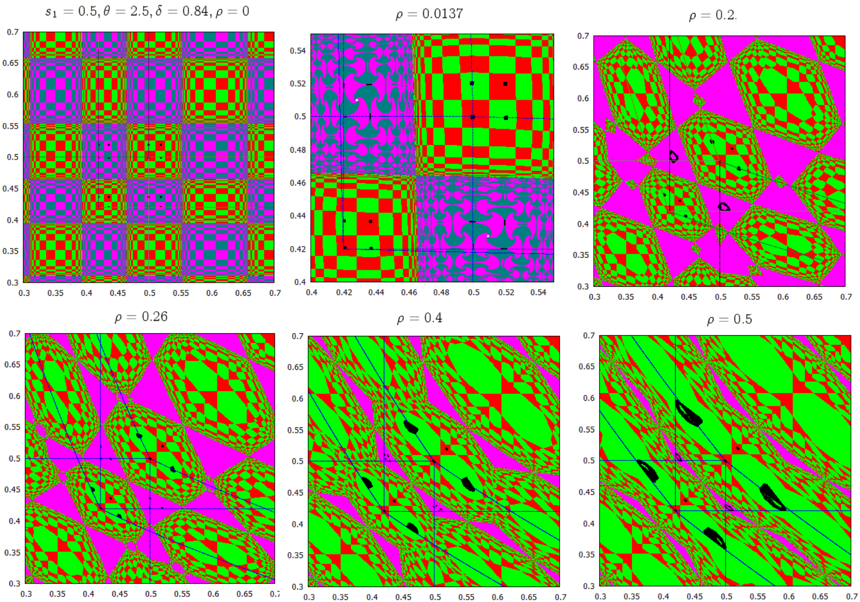
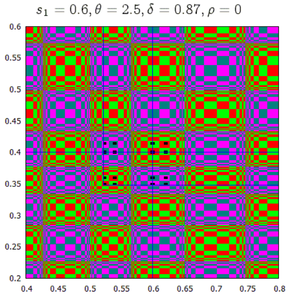


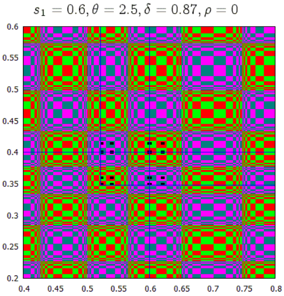
Coupled system: symmetric case (2)

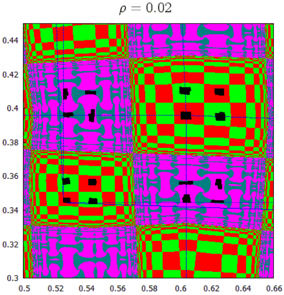


Coupled system: increasing ρ in asymmetric case

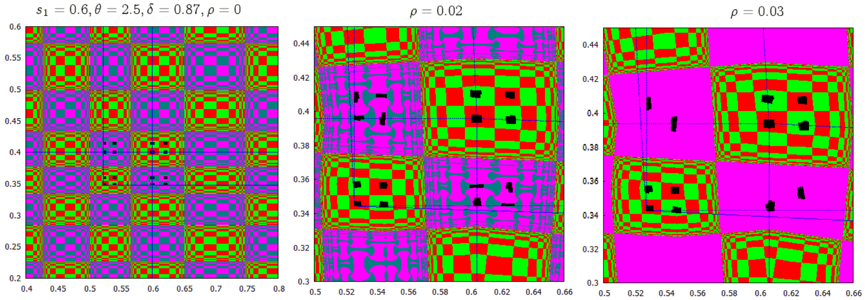


Coupled system: increasing ρ in asymmetric case

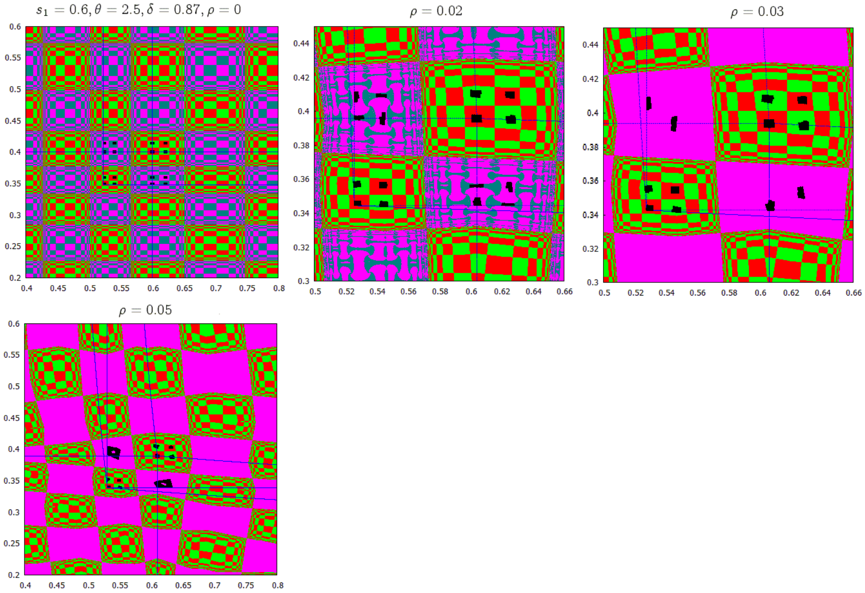




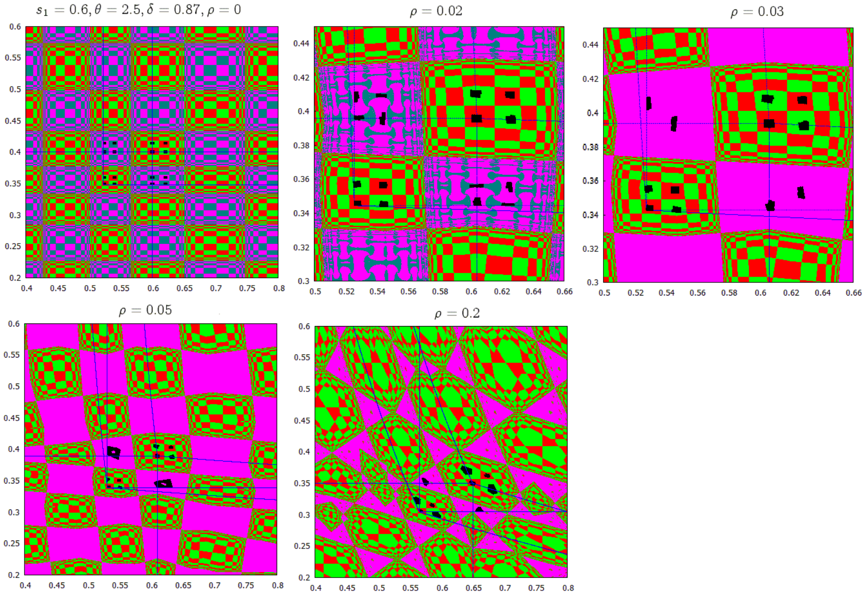
Coupled system: increasing ρ in asymmetric case



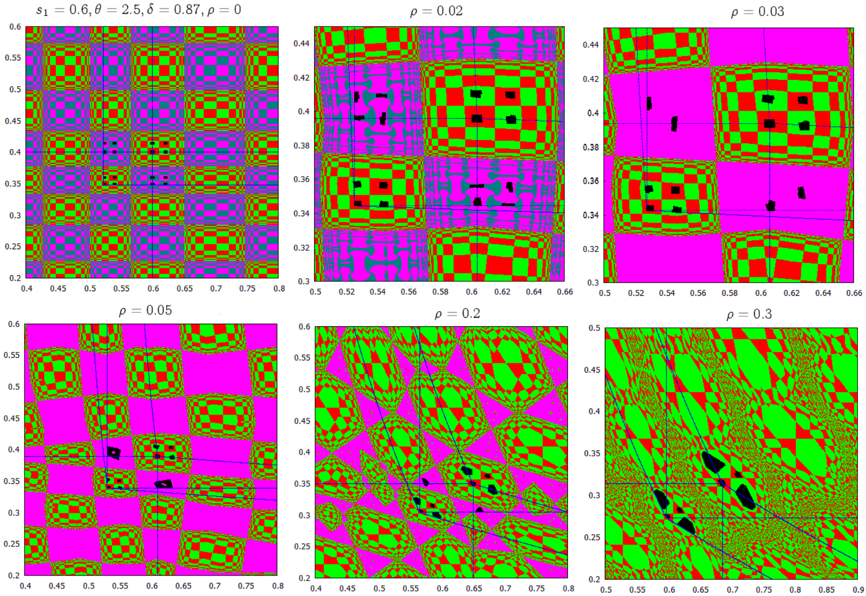
Coupled system: increasing ρ in asymmetric case



Coupled system: increasing ρ in asymmetric case



Coupled system: increasing ρ in asymmetric case



Outline of the research project

- Description of basin transformations related to homoclinic bifurcations (fractalization of the basin boundary) and to contacts of basin boundaries with critical lines;
- Transformations of chaotic attractors related to merging, expantion and final bifurcations;
- Role of border collision bifurcations in the observed bifurcation scenarious.

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