Selection and Sorting of Heterogeneous Firms Through Competitive Pressures

Kiminori Matsuyama  
Northwestern University

Philip Ushchev  
HSE University, St. Petersburg

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**Competitive Pressures on Heterogeneous Firms**

How do competitive pressures affect selection of firms with different productivity? Or sorting across different markets?

- Melitz (2003): monopolistic competition (MC) with heterogeneous firms under CES
  - MC firms sell their products at an exogenous & common markup rate, *unresponsive to competitive pressures*
  - Market size: no effect on distribution of firm types and on their behaviors; All adjustments at the *extensive margin.*
- Melitz-Ottaviano (2008) depart from CES using *Linear DS + the outside competitive sector*

We depart from CES using **H.S.A. (Homothetic with a Single Aggregator)** DS with gross substitutes

- **Homothetic** (unlike the linear DS and most other commonly-used non-CES DSs)
  - a single measure of market size; the demand composition does not matter.
  - isolate the effect of endogenous markup rate from nonhomotheticity
  - straightforward to use it as a building block for multi-sector GE models
- **Nonparametric** and **flexible** (unlike CES and translog, which are special cases)
  - can be used to perform robustness-check for CES and translog
  - allow for (but no need to impose) the choke price, Marshall’s 2\textsuperscript{nd} law as well as *what we call* the 3\textsuperscript{rd} law
- **Tractable** due to **Single Aggregator** (unlike Kimball, which needs two aggregators), a *sufficient statistic* for competitive pressures, which acts like a *magnifier of firm heterogeneity*. A simple diagram for
  - proving the existence & the uniqueness of free-entry equilibrium with firm heterogeneity
  - conducting most comparative statics without *parametric* restrictions on the demand or productivity distribution.
    - e.g., no need to assume zero overhead cost (unlike MO and ACDR)
- Defined by **the market share function**, for which data is readily available and easily identifiable.
Symmetric H.S.A. (Homothetic with a Single Aggregator) with Gross Substitutes

Here we consider a continuum of varieties ($\omega \in \Omega$), gross substitutes, and symmetry (Our 2017 paper for a general analysis).

Market Share of $\omega \in \Omega$ depends solely on its single relative price (= its own price/the common price aggregator)

$$\frac{p_{\omega}x_{\omega}}{px} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_{\omega}} = s \left( \frac{p_{\omega}}{A(\mathbf{p})} \right),$$

where

$$\int_{\Omega} s \left( \frac{p_{\omega}}{A(\mathbf{p})} \right) d\omega \equiv 1.$$

1. $s: \mathbb{R}_+^+ \rightarrow \mathbb{R}_+$: the market share function, decreasing in the relative price for $s(z) > 0$ with $\lim_{z \to z^\text{c}} s(z) = 0$.
   - If $z^\text{c} \equiv \inf\{z > 0|s(z) = 0\} < \infty$, $z^\text{c}A(\mathbf{p})$ is the choke price.

2. $A(\mathbf{p})$: the common price aggregator defined implicitly by the adding-up constraint $\int_{\Omega} s(p_{\omega}/A)d\omega \equiv 1.$

By construction, market shares add up to one; $A(\mathbf{p})$ linear homogenous in $\mathbf{p}$ for a fixed $\Omega$. A larger $\Omega$ reduces $A(\mathbf{p})$.

- CES if $s(z) = \gamma z^{1-\sigma}, (\sigma > 1)$; translog cost if $s(z) = -\gamma \ln \left( \frac{z}{z^\text{c}} \right)$; CoPaTh if $s(z) = \gamma \left[ 1 - \left( \frac{z}{z^\text{c}} \right)^{1-\rho} \right]^{\frac{\rho}{1-\rho}}, (0 < \rho < 1)$.

Unit Cost Function: $P(\mathbf{p}) \propto A(\mathbf{p}) \exp \left\{ -\int_{\Omega} \left[ \int_{p_{\omega}/A(\mathbf{p})}^{z^\text{c}} \frac{s(\xi)}{\xi} d\xi \right] d\omega \right\}$

Note: Our 2017 paper proved that $P(\mathbf{p})$ is quasi-concave and that $P(\mathbf{p})/A(\mathbf{p}) \neq c$ for any $c > 0$, unless CES

- $A(\mathbf{p})$, the inverse measure of competitive pressures, fully captures cross price effects in the demand system
- $P(\mathbf{p})$, the inverse measure of TFP, captures the productivity consequences of price changes
Monopolistic Competition under H.S.A.: Pricing

Pricing (Lerner) Formula
\[ p_\psi \left[ 1 - \frac{1}{\zeta(p_\psi/A)} \right] = \psi \implies \frac{p_\psi}{A} \left[ 1 - \frac{1}{\zeta(p_\psi/A)} \right] = \frac{\psi}{A} \]

where
\[ \zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} \equiv 1 - \varepsilon_s(z) > 1, \quad \text{for } z \in (0, \bar{z}); \quad \lim_{z \to \bar{z}} \zeta(z) = -\lim_{z \to \bar{z}} \varepsilon_s(z) = \infty, \text{if } \bar{z} < \infty. \]

\( \psi \): firm-specific marginal cost (in labor, the numeraire)

\( A = A(p) \): the inverse measure of competitive pressures, common across firms, a sufficient statistic.

Relative price
\[ z_\psi \equiv \frac{p_\psi}{A} = Z\left(\frac{\psi}{A}\right), \quad \text{an increasing function of } \psi/A, \text{ the normalized cost, only.} \]

Price elasticity
\[ \zeta\left(Z\left(\frac{\psi}{A}\right)\right) \equiv \sigma\left(\frac{\psi}{A}\right) > 1 \]

Markup rate
\[ \mu_\psi \equiv \frac{p_\psi}{\psi} = \frac{\sigma(\psi/A)}{\sigma(\psi/A) - 1} \equiv \mu\left(\frac{\psi}{A}\right) > 1 \]

Pass-through rate
\[ \rho_\psi \equiv \frac{\partial \ln p_\psi}{\partial \ln \psi} = \frac{d \ln Z(\psi/A)}{d \ln (\psi/A)} \equiv \varepsilon_Z\left(\frac{\psi}{A}\right) \equiv \rho\left(\frac{\psi}{A}\right) = 1 + \varepsilon_\mu\left(\frac{\psi}{A}\right) \]

are all functions of \( \psi/A \) only, continuously differentiable under mild regularity conditions.

More competitive pressures, a lower \( A \), act like a magnifier of firm heterogeneity.
Monopolistic Competition under H.S.A.: Revenue, Profit, & Employment

**Revenue**

\[ R_\psi = s(z_\psi)L = s\left(\frac{Z(\psi)}{A}\right)L \equiv r\left(\frac{\psi}{A}\right)L \quad \Rightarrow \quad \varepsilon_r\left(\frac{\psi}{A}\right) = -\left[\sigma\left(\frac{\psi}{A}\right) - 1\right]\rho\left(\frac{\psi}{A}\right) < 0 \]

**Gross Profit**

\[ \Pi_\psi = \frac{s(z_\psi)}{\zeta(z_\psi)}L = \frac{r(\psi/A)}{\sigma(\psi/A)}L \equiv \pi\left(\frac{\psi}{A}\right)L \quad \Rightarrow \quad \varepsilon_\pi\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right) < 0 \]

**Variable Employment**

\[ \psi_x = R_\psi - \Pi_\psi = \frac{r(\psi/A)}{\mu(\psi/A)}L \equiv \ell\left(\frac{\psi}{A}\right)L \quad \Rightarrow \quad \varepsilon_\ell\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right) \leq 0 \]

- Revenue \( r(\psi/A)L \), profit \( \pi(\psi/A)L \), employment \( \ell(\psi/A)L \), all functions of \( \psi/A \), multiplied by market size \( L \), continuously differentiable under mild regularity conditions.
- Market size affects the relative profit, revenue, and employment across firms only through its effects on \( A \).
- Both revenue \( r(\psi/A)L \) and profit \( \pi(\psi/A)L \) are always strictly decreasing in \( \psi/A \).
- Employment \( \ell(\psi/A)L \) may be nonmonotonic in \( \psi/A \).
  - If the markup rate declines with \( \psi/A \), employment cannot decline as fast as the revenue.
  - If the markup rate declines faster than the revenue, the employment is increasing in \( \psi/A \).

Again, more competitive pressures, a lower \( A \), act like a magnifier of firm heterogeneity.
General Equilibrium: Existence and Uniqueness

As in Melitz, firms pay the entry cost $F_e > 0$ to draw $\psi \sim G(\psi)$, cdf with the support, $(\underline{\psi}, \overline{\psi}) \subset (0, \infty)$, and pay the overhead $F > 0$ to stay & produce.

Cutoff Rule: stay if $\psi < \psi_c$; exit if $\psi > \psi_c$, where

$$\pi \left( \frac{\psi_c}{A} \right) L = F$$

positively-sloped $A \downarrow$ (more competitive pressures) $\implies \psi_c \downarrow$ (tougher selection)

Free Entry Condition:

$$F_e = \int_\psi^{\psi_c} \left[ \pi \left( \frac{\psi}{A} \right) L - F \right] dG(\psi)$$

negative-sloped, both $A \downarrow$ (more competitive pressures) and $\psi_c \downarrow$ (tougher selection) make entry less attractive.

$A = A(p)$ and $\psi_c$: uniquely determined, respond continuously to $F_e/L$ & $F/L$ under mild regularity conditions. (This proof of the unique existence applies also to the Melitz model under CES.)

With $A$ and $\psi_c$ fixed, the adding-up constraint pins down the mass of entrants, $M$ and that of active firms, $MG(\psi_c)$. 
Cross-Sectional Implications of Marshall’s 2nd Law

(A2): \( \zeta(z_{\psi}) \) is increasing in \( z_{\psi} \equiv p_{\psi}/A = Z(\psi/A) \)

- **Price elasticity** \( \zeta(Z(\psi/A)) \equiv \sigma(\psi/A) \) increasing in \( \psi/A \); high-\( \psi \) firms operate at more elastic parts of demand curve.

  - **Markup Rate**, \( \mu(\psi/A) \), decreasing in \( \psi/A \) \( \Leftrightarrow \) \( \varepsilon_{\mu}(\psi/A) < 0 \)
    high-\( \psi \) firms charge lower markup rates.

  - **Incomplete Pass-Through**: The pass-through rate, \( \rho(\psi/A) = 1 + \varepsilon_{\mu}(\psi/A) < 1 \).

- **Procompetitive effect of entry/Strategic complementarity in pricing**, \( \frac{\partial \ln p_{\psi}}{\partial \ln A} = 1 - \rho(\psi/A) > 0 \).
  Firms set the price lower under more competitive pressures \( (A = A(p) \downarrow) \), due to either a larger \( \Omega \) and/or a lower \( p \)

- **Profit**, \( \pi(\psi/A)L \), always decreasing, **strictly log-supermodular** in \( \psi \) and \( A \).
  \( A \downarrow \rightarrow \) a proportionately larger decline in profit for high-\( \psi \) firms \( \rightarrow \) Larger dispersion of profit

\( f(\psi/A) \) is (strictly) log-super(sub)modular in \( \psi \& A \) \( \Leftrightarrow \) \( \varepsilon_{f} \left( \frac{\psi}{A} \right) \equiv \frac{d \ln f(\psi/A)}{d \ln(\psi/A)} \) is (strictly) decreasing (increasing) in \( \psi/A \).
Cross-Sectional Implications of the 3rd Law

In addition to A2, if we further assume, with some empirical support,

\[ \rho(\psi/A) = 1 + \varepsilon_\mu(\psi/A) \] is weak(strictly) increasing--we call it \textbf{Weak(Strong) 3rd Law}.

Under translog, \( \rho(\psi/A) \) is strictly decreasing, violating A3

- **Markup rate**, \( \mu(\psi/A) \), decreasing under A2, \textbf{log-submodular} in \( \psi \) & \( A \) under A3. For strong A3, it is strict and \( A \downarrow \rightarrow \) a proportionately smaller decline in markup rate for high-\( \psi \) firms \( \rightarrow \) Smaller dispersion of markup rate

- **Revenue**, \( r(\psi/A)L \), always decreasing, \textbf{strictly log-supermodular} in \( \psi \) & \( A \) under weak A3
  \( A \downarrow \rightarrow \) a proportionately larger decline in revenue for high-\( \psi \) firms \( \rightarrow \) Larger dispersion of revenue

- **Employment**, \( \ell(\psi/A)L = \frac{r(\psi/A)}{\mu(\psi/A)}L \), \textit{hump-shaped} in \( \psi/A \), \textbf{strictly log-supermodular} in \( \psi \) & \( A \) under weak A3
  Employment increasing in \( \psi \) across all active firms with a large enough overhead/market size ratio.
  \( A \downarrow \rightarrow \) Employment up for the most productive firms.

- **Pass-through rate**, \( \rho(\psi/A) \), \textbf{strictly log-submodular} in \( \psi \) & \( A \) for a small enough \( \bar{z} \) under strong A3.
  \( A \downarrow \rightarrow \) a proportionately smaller increase in the pass-through rate for low-\( \psi \) firms among the active
Cross-Sectional Implications of More Competitive Pressures ($A \downarrow$)

**Profit Function:** $\Pi_\psi = \pi(\psi / A)L$
- *always* decreasing in $\psi$
- strictly log-supermodular *under A2*
  - $A \downarrow$ with $L$ fixed shifts down with a steeper slope at each $\psi$;
  - $A \downarrow$ due to $L \uparrow$, a parallel shift up, a *single-crossing*

- $\ln \Pi_\psi = \ln \pi \left( \frac{\psi}{A} \right) + \ln L$
- $\ln \psi$
- $\ln \Psi$

**Markup Rate Function:** $\mu_\psi = \mu(\psi / A) > 1$
- *decreasing in* $\psi$ *under A2*
- weakly log-submodular *under Weak A3*
- strictly log-submodular *under Strong A3*
  - $A \downarrow$ shifts down with a flatter slope at each $\psi$

- $\ln \mu_\psi = \ln \mu \left( \frac{\psi}{A} \right) > 0$
- $\ln \psi$
- $\ln \Psi$

✓ With $\ln \psi$ in the horizontal axis, $A \downarrow$ causes a parallel leftward shift of the graphs.
✓ $f(\psi / A)$ is (strictly) log-super(sub)modular in $\psi$ & $A \Leftrightarrow \ln f(\psi / A)$ is (strictly) concave( convex) in $\ln(\psi / A)$.

Under Weak A3, $R_\psi = r(\psi / A)L$, strictly log-supermodular and shares similar properties with $\pi(\psi / A)L$. 
Employment Function: $\ell(\psi/A)L = r(\psi/A)L/\mu(\psi/A)$
- *Hump-shaped* in $\psi$ under $A2$ and weak $A3$.
  $\rightarrow A \downarrow$ shifts up (down) for a low (high) $\psi$ with $A \downarrow$
- Strictly log-supermodular *under Weak A3*
  for $A \downarrow$ with a fixed $L$; for $A \downarrow$ caused by $L \uparrow$
  *Single-crossing* even with a fixed $L$

Pass-Through Rate Function: $\rho_\psi = \rho(\psi/A)$
- $\rho(\psi/A) < 1$ *under A2*, hence it cannot be strictly log-submodular for a higher range of $\psi/A$
- Strictly increasing in $\psi$ *under Strong A3*
- Strictly log-submodular for a lower range of $\psi/A$ *under A2 and Strong A3* $\Rightarrow A \downarrow$ shifts up with a steeper slope at each $\psi$ with a small enough $\bar{z}$.

In sum, more competitive pressures ($A \downarrow$)
$\rightarrow \mu(\psi/A) \downarrow$ under $A2$ & $\rho(\psi/A) \uparrow$ under strong $A3$
$\rightarrow$ Profit, Revenue, Employment become more concentrated among the most productive.
GE Comparative Statics Implications: Selection (in a single market setting)

Effects of $F_e \downarrow$

Effects of $L \uparrow$ if $\sigma'(\cdot) > 0$ (i.e., A2)

Effects of $F \downarrow$ if $\ell'(\cdot) > 0$

- $L \uparrow$ under A2: the profit up for low-$\psi$ and down for high-$\psi$. (Similarly on the revenue under A2 and the weak A3)
- All 3 cases lead to $\psi_c \downarrow$ & $A \downarrow$, creating a non-trivial composition effect
  - Under A2, $A \downarrow$ causes $\mu(\psi/A) \downarrow$ for each $\psi$, but $\psi_c \downarrow$ means high-$\psi$ firms with lower $\mu(\psi/A)$ drop out.
  - Under strong A3, $A \downarrow$ causes $\rho(\psi/A) \uparrow$ for each $\psi$, but $\psi_c \downarrow$ means high-$\psi$ firms with higher $\rho(\psi/A)$ drop out.

The average markup (or pass-through) rate can go either way, with $F_e \downarrow$ + Pareto-productivity a knife-edge case

More competition, which causes more concentration, may result in the rise of markup.

- The effects on $M$ & $MG(\psi_c)$ depend on whether $E_G(\psi) = \psi g(\psi)/G(\psi)$ is decreasing, constant, or increasing.
GE Implications: Sorting (in a multi-market setting)

More competitive pressures in larger markets:
\[ L_1 > L_2 > \cdots > L_J > 0 \Rightarrow 0 < A_1 < A_2 < \cdots < A_J < \infty \]
Under A2, more efficient firms sort themselves into larger markets
Firms \( \psi \in (\psi_{j-1}, \psi_j) \) entering market- \( j \)

Markup Rate across markets under A2

Pass-Through Rate across markets under strong A3

The Composition Effect: examples with Pareto-productivity such that
- The average markup rates higher (the average pass-through rates lower under Strong A3) in larger (more competitive) markets
- A decline in \( F_e \) causes uniform declines in \( \psi_j \) & \( A_j \) with the average markup/pass-through rates unchanged.

A caution against testing A2/A3 by comparing the average markup/pass-through rates in cross-section of cities.
(Highly Selective) Literature Review

**H.S.A. Demand System:** Matsuyama-Ushchev (2017)
**MC with Heterogeneous Firms:** Melitz (2003) and many others: Melitz-Redding (2015) for a survey
**MC under non-CES demand systems:** Thisse-Ushchev (2018) for a survey
- **Nonhomothetic non-CES:**
  - $U = \int_{\Omega} u(x_\omega) d\omega$: Dixit-Stiglitz (77), Behrens-Murata (07), ZKPT (12), Mrázová-Neary(17), Dhingra-Morrow (19); ACDR (19)
- **Homothetic non-CES:** Feenstra (2003), Kimball (1995), Matsuyama-Ushchev (2020a, 2020b, 2022)

**Empirical Evidence:** *The 2nd Law:* DeLoecker-Goldberg (14), Burstein-Gopinath (14); *The 3rd Law:* Berman et.al.(12); Amiti et.al. (19); *Market Size Effects:* Campbell-Hopenhayn(05); *Rise of markup:* Autor et.al.(20), DeLoecker et.al.(20)

**Selection of Heterogeneous Firms through Competitive Pressures**
Melitz-Ottaviano (2008), Baqee-Fahri-Sangani (2021)

**Sorting of Heterogeneous Firms Across Markets in General Equilibrium**

**Sorting of Heterogeneous Firms Across Markets in Reduced Form/Partial Equilibrium**

**Log-Super(Sub)modularity:** Costinot (2009), Costinot-Vogel (2010, 2015)