Selection and Sorting of Heterogeneous Firms  
Through Competitive Pressures

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Competitive Pressures on Heterogeneous Firms

How do competitive pressures affect selection of firms with different productivity? Or sorting across different markets?

- Melitz (2003): monopolistic competition (MC) with heterogeneous firms under CES
  - MC firms sell their products at an exogenous & common markup rate, *unresponsive to competitive pressures*
  - Market size: no effect on distribution of firm types and on their behaviors; All adjustments at *the extensive margin*.

- Melitz-Ottaviano (2008) depart from CES using *Linear DS + the outside competitive sector*

We depart from CES using **H.S.A. (Homothetic with a Single Aggregator)** DS with gross substitutes

- **Homothetic** (unlike the linear DS and most other commonly-used non-CES DSs)
  - a single measure of market size; the demand composition does not matter.
  - isolate the effect of endogenous markup rate from nonhomotheticity
  - straightforward to use it as a building block for multi-sector GE models

- **Nonparametric** and **flexible** (unlike CES and translog, which are special cases)
  - can be used to perform robustness-check for CES and translog
  - allow for (but no need to impose) the choke price, Marshall’s 2nd law as well as *what we call* the 3rd law

- **Tractable** due to **Single Aggregator** (unlike Kimball, which needs two aggregators), a *sufficient statistic* for competitive pressures, which acts like a *magnifier of firm heterogeneity*. A simple diagram for
  - proving the existence & the uniqueness of free-entry equilibrium with firm heterogeneity
  - conducting most comparative statics without *parametric* restrictions on the demand or productivity distribution.
  - e.g., no need to assume zero overhead cost (unlike MO and ACDR)

- Defined by **the market share function**, for which data is readily available and easily identifiable.
Symmetric H.S.A. (Homothetic with a Single Aggregator) with Gross Substitutes

Here we consider a continuum of varieties ($\omega \in \Omega$), gross substitutes, and symmetry (Our 2017 paper for a general analysis)

Market Share of $\omega \in \Omega$ depends solely on its single relative price (= its own price/the common price aggregator)

$$\frac{p_{\omega}x_{\omega}}{p_{x}} = \frac{\partial \ln P(p)}{\partial \ln p_{\omega}} = s\left(\frac{p_{\omega}}{A(p)}\right),$$

where

$$\int_{\Omega} s\left(\frac{p_{\omega}}{A(p)}\right) d\omega \equiv 1.$$

- $s: \mathbb{R}^{++} \rightarrow \mathbb{R}^{+}$: the market share function, decreasing in the relative price for $s(z) > 0$ with $\lim_{z \to \bar{z}} s(z) = 0$.
  - If $\bar{z} \equiv \inf\{z > 0 | s(z) = 0\} < \infty$, $\bar{z}A(p)$ is the choke price.

- $A(p)$: the common price aggregator defined implicitly by the adding-up constraint

$$\int_{\Omega} s\left(\frac{p_{\omega}}{A(p)}\right) d\omega \equiv 1.$$

By construction, market shares add up to one; $A(p)$ linear homogenous in $p$ for a fixed $\Omega$. A larger $\Omega$ reduces $A(p)$.

CES if $s(z) = \gamma z^{1-\sigma}, (\sigma > 1)$; translog cost if $s(z) = -\gamma \ln \left(\frac{z}{\bar{z}}\right)$; CoPaTh if $s(z) = \gamma \left[1 - \left(\frac{z}{\bar{z}}\right)^{1-\rho}\right]^{\frac{1-\rho}{\rho}}, (0 < \rho < 1)$.

Unit Cost Function: $P(p) \propto A(p) \exp\left\{-\int_{\Omega} \left[\int_{p_{\omega}/A(p)}^{z} \frac{s(\xi)}{\xi} d\xi\right] d\omega\right\}$

Note: Our 2017 paper proved that $P(p)$ is quasi-concave and that $P(p)/A(p) \neq c$ for any $c > 0$, unless CES

✓ $A(p)$, the inverse measure of competitive pressures, fully captures cross price effects in the demand system

✓ $P(p)$, the inverse measure of TFP, captures the productivity consequences of price changes
Monopolistic Competition under H.S.A.: Pricing

**Pricing (Lerner) Formula**

\[ p \psi \left[ 1 - \frac{1}{\zeta(p\psi / A)} \right] = \psi, \]

where

\[ \zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} \equiv 1 - \varepsilon_s(z) > 1, \quad \text{for } z \in (0, \bar{z}); \lim_{z \to \bar{z}} \zeta(z) = -\lim_{z \to \bar{z}} \varepsilon_s(z) = \infty, \text{if } \bar{z} < \infty. \]

\( \psi \): firm-specific marginal cost (in labor, the numeraire)

\( A = A(p) \): the inverse measure of **competitive pressures**, common across firms, a sufficient statistic.

**Relative price**

\[ z_\psi \equiv \frac{p\psi}{A} = Z \left( \frac{\psi}{A} \right), \quad \text{an increasing function of } \psi / A, \text{ the normalized cost, only.} \]

**Price elasticity**

\[ \zeta \left( Z \left( \frac{\psi}{A} \right) \right) \equiv \sigma \left( \frac{\psi}{A} \right) > 1 \]

**Markup rate**

\[ \mu_\psi \equiv \frac{p\psi}{\psi} = \frac{\sigma(\psi/A)}{\sigma(\psi/A) - 1} \equiv \mu \left( \frac{\psi}{A} \right) > 1 \]

**Pass-through rate**

\[ \rho_\psi \equiv \frac{\partial \ln p\psi}{\partial \ln \psi} \equiv \frac{d \ln Z(\psi/A)}{d \ln (\psi/A)} \equiv \varepsilon_z \left( \frac{\psi}{A} \right) \equiv \rho \left( \frac{\psi}{A} \right) = 1 + \varepsilon_\mu \left( \frac{\psi}{A} \right) \]

are all functions of \( \psi / A \) only, continuously differentiable under mild regularity conditions.

More competitive pressures, a lower \( A \), act like a magnifier of firm heterogeneity.
Monopolistic Competition under H.S.A.: Revenue, Profit, & Employment

Revenue
\[ R_\psi = s(z_\psi) L = s\left(Z\left(\frac{\psi}{A}\right)\right) L \equiv r\left(\frac{\psi}{A}\right) L \implies \mathcal{E}_r\left(\frac{\psi}{A}\right) = -\left[\sigma\left(\frac{\psi}{A}\right) - 1\right] \rho\left(\frac{\psi}{A}\right) < 0 \]

(Gross) Profit
\[ \Pi_\psi = \frac{S(z_\psi)}{\xi(z_\psi)} L = \frac{r(\psi/A)}{\sigma(\psi/A)} L \equiv \pi\left(\frac{\psi}{A}\right) L \implies \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right) < 0 \]

(Variable) Employment
\[ \psi x_\psi = R_\psi - \Pi_\psi = \frac{r(\psi/A)}{\mu(\psi/A)} L \equiv \ell\left(\frac{\psi}{A}\right) L \implies \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right) \rho\left(\frac{\psi}{A}\right) \leq 0 \]

- Revenue \( r(\psi/A) L \), profit \( \pi(\psi/A) L \), employment \( \ell(\psi/A) L \), all functions of \( \psi/A \), multiplied by market size \( L \), continuously differentiable under mild regularity conditions.
- Market size affects the relative profit, revenue, and employment across firms only through its effects on \( A \).
- Both revenue \( r(\psi/A) L \) and profit \( \pi(\psi/A) L \) are always strictly decreasing in \( \psi/A \).
- Employment \( \ell(\psi/A) L \) may be nonmonotonic in \( \psi/A \).
  - If the markup rate declines with \( \psi/A \), employment cannot decline as fast as the revenue.
  - If the markup rate declines faster than the revenue, the employment is increasing in \( \psi/A \).

Again, more competitive pressures, a lower \( A \), act like a magnifier of firm heterogeneity.
General Equilibrium: Existence and Uniqueness

As in Melitz, firms pay the entry cost $F_e > 0$ to draw $\psi \sim G(\psi)$, cdf with the support, $(\underline{\psi}, \overline{\psi}) \subset (0, \infty)$, and pay the overhead $F > 0$ to stay & produce.

**Cutoff Rule:** stay if $\psi < \psi_c$; exit if $\psi > \psi_c$, where

$$\pi \left( \frac{\psi_c}{A} \right) L = F$$

*positively-sloped* $A \downarrow$ (more competitive pressures) $\Rightarrow$ $\psi_c \downarrow$ (tougher selection)

**Free Entry Condition:**

$$F_e = \int_{\underline{\psi}}^{\psi_c} \pi \left( \frac{\psi}{A} \right) L - F \, dG(\psi)$$

*negative-sloped*, both $A \downarrow$ (more competitive pressures) and $\psi_c \downarrow$ (tougher selection) make entry less attractive.

$A = A(\mathbf{p})$ and $\psi_c$: uniquely determined, respond continuously to $F_e/L$ & $F/L$ under mild regularity conditions. (This proof of the unique existence applies also to the Melitz model under CES.)

With $A$ and $\psi_c$ fixed, the adding-up constraint pins down the mass of entrants, $M$ and that of active firms, $MG(\psi_c)$.
Cross-Sectional Implications of Marshall’s 2nd Law

(A2): $\zeta(z_\psi)$ is increasing in $z_\psi \equiv p_\psi / A = Z(\psi/A)$

- **Price elasticity** $\zeta(Z(\psi/A)) \equiv \sigma(\psi/A)$ increasing in $\psi/A$; high-$\psi$ firms operate at more elastic parts of demand curve.
  - **Markup Rate**, $\mu(\psi/A)$, decreasing in $\psi/A \Leftrightarrow \mathcal{E}_\mu(\psi/A) < 0$
    - high-$\psi$ firms charge lower markup rates.
  - **Incomplete Pass-Through**: The pass-through rate, $\rho(\psi/A) = 1 + \mathcal{E}_\mu(\psi/A) < 1$.

- **Procompetitive effect of entry/Strategic complementarity in pricing**,\[\frac{\partial \ln p_\psi}{\partial \ln A} = 1 - \rho(\psi/A) > 0.\]
  Firms set the price lower under more competitive pressures ($A = A(p) \downarrow$), due to either a larger $\Omega$ and/or a lower $p$.

- **Profit**, $\pi(\psi/A)L$, always decreasing, *strictly log-supermodular* in $\psi$ and $A$.
  $A \downarrow \rightarrow$ a proportionately larger decline in profit for high-$\psi$ firms $\rightarrow$ Larger dispersion of profit

$\checkmark f(\psi/A)$ is (strictly) log-super(sub)modular in $\psi$ & $A \Leftrightarrow \mathcal{E}_f \left(\frac{\psi}{A}\right) \equiv \frac{d \ln f(\psi/A)}{d \ln(\psi/A)}$ is (strictly) decreasing (increasing) in $\psi/A$. 
Cross-Sectional Implications of the 3\textsuperscript{rd} Law

In addition to A2, if we further assume, with some empirical support,

\[(A3): \rho(\psi/A) = 1 + \varepsilon \mu(\psi/A) \text{ is weak(strictly) increasing--we call it Weak(Strong) 3\textsuperscript{rd} Law.}\]

Under translog, \(\rho(\psi/A)\) is strictly decreasing, violating A3

- **Markup rate**, \(\mu(\psi/A)\), decreasing under A2, **log-submodular** in \(\psi \& A\) under A3. For strong A3, it is strict and \(A \downarrow \rightarrow \) a proportionately smaller decline in markup rate for high-\(\psi\) firms \(\rightarrow\) Smaller dispersion of markup rate

- **Revenue**, \(r(\psi/A)L\), always decreasing, **strictly log-supermodular** in \(\psi \& A\) under weak A3
  \(A \downarrow \rightarrow\) a proportionately larger decline in revenue for high-\(\psi\) firms \(\rightarrow\) Larger dispersion of revenue

- **Employment**, \(\ell(\psi/A)L = \frac{r(\psi/A)}{\mu(\psi/A)} L\), **hump-shaped** in \(\psi/A\), **strictly log-supermodular** in \(\psi \& A\) under weak A3
  Employment increasing in \(\psi\) across all active firms with a large enough overhead/market size ratio.
  \(A \downarrow \rightarrow\) Employment up for the most productive firms.

- **Pass-through rate**, \(\rho(\psi/A)\), **strictly log-submodular** in \(\psi \& A\) for a small enough \(\bar{Z}\) under strong A3.
  \(A \downarrow \rightarrow\) a proportionately smaller increase in the pass-through rate for low-\(\psi\) firms among the active
Cross-Sectional Implications of More Competitive Pressures ($A \downarrow$)

**Profit Function:** $\Pi_\psi = \pi(\psi / A)L$
- *always* decreasing in $\psi$
- strictly log-supermodular *under* $A2$
  - $\rightarrow A \downarrow$ with $L$ fixed shifts down with a steeper slope at each $\psi$;
  - $\rightarrow A \downarrow$ due to $L \uparrow$, a parallel shift up, *a single-crossing*

\[
\ln \Pi_\psi = \ln \pi \left( \frac{\psi}{A} \right) + \ln L
\]

**Markup Rate Function:** $\mu_\psi = \mu(\psi / A) > 1$
- *decreasing* in $\psi$ *under* $A2$
- weakly log-submodular *under* *Weak* $A3$
- strictly log-submodular *under* *Strong* $A3$
  - $\rightarrow A \downarrow$ shifts down with a flatter slope at each $\psi$

\[
\ln \mu_\psi = \ln \mu \left( \frac{\psi}{A} \right) > 0
\]

- With $\ln \psi$ in the horizontal axis, $A \downarrow$ causes a parallel leftward shift of the graphs.
- $f(\psi/A)$ is (strictly) log-super(sub)modular in $\psi$ & $A \Leftrightarrow \ln f(\psi/A)$ is (strictly) concave( convex) in $\ln(\psi/A)$.

Under *Weak* A3, $R_\psi = r(\psi/A)L$, strictly log-supermodular and shares similar properties with $\pi(\psi/A)L$. 

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**Employment Function:** $\ell(\psi / A)L = r(\psi / A)L / \mu(\psi / A)$

- **Hump-shaped** in $\psi$ under $A_2$ and weak $A_3$.
  $\rightarrow A \downarrow$ shifts up (down) for a low (high) $\psi$ with $A \downarrow$
- **Strictly log-supermodular** under Weak $A_3$
  for $A \downarrow$ with a fixed $L$; for $A \downarrow$ caused by $L \uparrow$

**Pass-Through Rate Function:** $\rho_{\psi} = \rho(\psi / A)$

- $\rho(\psi / A) < 1$ under $A_2$, hence it cannot be strictly log-submodular for a higher range of $\psi / A$
- **Strictly increasing in $\psi$ under Strong $A_3$**
- **Strictly log-submodular** for a lower range of $\psi / A$ under $A_2$ and Strong $A_3$ $\Rightarrow A \downarrow$ shifts up with a steeper slope at each $\psi$ with a small enough $\bar{z}$.

In summary, more competitive pressures ($A \downarrow$)
$\rightarrow \mu(\psi / A) \downarrow$ under $A_2$ & $\rho(\psi / A) \uparrow$ under strong $A_3$
$\rightarrow$ Profit, Revenue, Employment become more concentrated among the most productive.
GE Comparative Statics Implications: Selection (in a single market setting)

Effects of $F_e \downarrow$

Effects of $L \uparrow$ if $\sigma'(\cdot) > 0$ (i.e., A2)

Effects of $F \downarrow$ if $\ell'(\cdot) > 0$

- $L \uparrow$ under A2: the profit up for low-$\psi$ and down for high-$\psi$. (Similarly on the revenue under A2 and the weak A3)
- All 3 cases lead to $\psi_c \downarrow$ & $A \downarrow$, creating a non-trivial composition effect
  - Under A2, $A \downarrow$ causes $\mu(\psi/A) \downarrow$ for each $\psi$, but $\psi_c \downarrow$ means high-$\psi$ firms with lower $\mu(\psi/A)$ drop out.
  - Under strong A3, $A \downarrow$ causes $\rho(\psi/A) \uparrow$ for each $\psi$, but $\psi_c \downarrow$ means high-$\psi$ firms with higher $\rho(\psi/A)$ drop out.

The average markup (or pass-through) rate can go either way, with $F_e \downarrow$ + Pareto-productivity a knife-edge case

More competition, which causes more concentration, may result in the rise of markup.

- The effects on $M$ & $MG(\psi_c)$ depend on whether $\mathcal{E}_G(\psi) \equiv \psi g(\psi)/G(\psi)$ is decreasing, constant, or increasing.
GE Implications: Sorting (in a multi-market setting)

More competitive pressures in larger markets:

\[ L_1 > L_2 > \cdots > L_J > 0 \Rightarrow 0 < A_1 < A_2 < \cdots < A_J < \infty \]

Under A2, more efficient firms sort themselves into larger markets with firms \( \psi \in (\psi_{j-1}, \psi_j) \) entering market- \( j \).

The composition effect: examples with Pareto-productivity such that

- The average markup rates higher (and the average pass-through rates lower under Strong A3) in larger (more competitive) markets
- A decline in \( F_e \) causes uniform declines in \( \psi_j \) & \( A_j \) with the average markup/pass-through rates unchanged.

A caution against testing A2/A3 by comparing the average markup/pass-through rates in cross-section of cities.
(Highly Selective) Literature Review

**H.S.A. Demand System:** Matsuyama-Ushchev (2017)

**MC with Heterogeneous Firms:** Melitz (2003) and many others: Melitz-Redding (2015) for a survey

**MC under non-CES demand systems:** Thisse-Ushchev (2018) for a survey

- **Nonhomothetic non-CES:**
  - $U = \int_{\Omega} u(x_\omega) d\omega$: Dixit-Stiglitz (77), Behrens-Murata (07), ZKPT (12), Mrázová-Neary (17), Dhingra-Morrow (19); ACDR (19)
  - **Linear-demand system:** Ottaviano-Tabuchi-Thisse (2002)

- **Homothetic non-CES:** Feenstra (2003), Kimball (1995), Matsuyama-Ushchev (2020a, 2020b, 2022)

**Empirical Evidence:** *The 2nd Law:* DeLoecker-Goldberg (14), Burstein-Gopinath (14); *The 3rd Law:* Berman et.al.(12); Amiti et.al. (19); *Market Size Effects:* Campbell-Hopenhayn(05); *Rise of markup:* Autor et.al.(20), DeLoecker et.al.(20)

**Selection of Heterogeneous Firms through Competitive Pressures**
Melitz-Ottaviano (2008), Baqee-Fahri-Sangani (2021)

**Sorting of Heterogeneous Firms Across Markets in General Equilibrium**

**Sorting of Heterogeneous Firms Across Markets in Reduced Form/Partial Equilibrium**

**Log-Super(Sub)modularity:** Costinot (2009), Costinot-Vogel (2010, 2015)