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362, Winter, 2006

Lectures #7 - 9: Exchange Rate Determination

In the previous lectures we learned that expectations about the long run matter a lot for determining what happens in the short run. The channel through which this operates is through E^e. So, to understand what happens in the short run when an exogenous variable changes value, we need to first understand the impact of the change on the long run, and hence on E^e. This is why we now begin with some models about the long run effects of various disturbances. After that, we turn to thinking about the short and long run.

1 Simple Monetary Approach to the Exchange Rate (Long Run)

Our simplest theory is the 'Simple Monetary Approach to the Exchange Rate'. The reason for this name is that it is simple, and the approach stresses the impact of monetary factors on the exchange rate. Later, we will develop more sophisticated approaches, which will allow us to think about the impact of other factors on the exchange rate too.

The monetary approach determines $R_{\$}$, E, and P_{US} in the long run using UIP ('uncovered interest parity'), MM (the money market equilibrium condition that says the supply of money must be equal to the demand for money), and PPP ('purchasing power parity'):

$$\begin{aligned}\text{UIP} &: \quad R_{\$} = R_f + \frac{E^e - E}{E} \\ \text{Money Market} &: \quad \frac{M}{P_{US}} = L(R_{\$}, Y). \\ \text{PPP} &: \quad \frac{P_f E}{P_{US}} = 1.\end{aligned}$$

Here, the subscript f denotes 'foreign'. According to our theory, every equation is satisfied at every date in the long run.

1. Equilibrium. In the above relationships, the superscript, e, on E means the value of the exchange rate, 'later'. Under the UIP, the differential between nominal interest rates in the US and abroad depends on the current interest rate and the interest rate expected to prevail later, when the interest rate payments are made.[1] For example, if the interest

[1] You should make sure you understand why this is so.

rate pays off in three months' time, then E^e refers to the interest rate three months in the future. We could think of applying the e superscript to other variables too. For example, M^e means the money supply three months later, in contrast with the money supply today, M. The money growth rate over the next three months is expected to be $(M^e - M)/M$. We can also think of applying the superscript, e, to the price level, so that P_{US}^e denotes the US price level in three months. The rate of inflation expected over the next three months is written $\pi^{US} = (P_{US}^e - P_{US})/P_{US}$. This superscript convention could be pushed even further. For example, we could let $E^{e,e}$ mean $(E^e)^e$, the value of the exchange rate *six* months later.

Now, the variables of the model are assumed to satisfy the above equations at *every* date in the long run. In particular, they should satisfy the equations three months from now:

$$\begin{aligned} \text{UIP} &: \quad R_\$^e = R_f^e + \frac{E^{e,e} - E^e}{E^e} \\ \text{Money Market} &: \quad \frac{M^e}{P_{US}^e} = L(R_\$^e, Y). \\ \text{PPP} &: \quad \frac{P_f^e E^e}{P_{US}^e} = 1. \end{aligned}$$

We will always suppose that interest rates are constant in the long run. That is, $R_\$^e = R_\$$, and $R_f^e = R_f$. Although we assume that $R_\$$ is a constant over time, what that constant value actually is, is determined by the model. In addition, for the most part we will assume that Y is constant, i.e., $Y^e = Y$. When we want to allow for the possibility that Y changes over time, we will suppose that it changes at a constant rate, i.e., that $(Y^e - Y)/Y$ is contant. We will assume that if the other exogenous variables, M, P_f, are changing, then their rate of change is constant in the long run too. That is, $(M^e - M)/M$ and $\pi_f = (P_f^e - P_f)/P_f$ are constant.

2. Properties of Equilibrium. Given the assumptions just stated, it is easy to see that UIP implies the rate of change in E, $(E^e - E)/E$, must be constant. The PPP equation then implies

$$\text{(PPP)}: \frac{E^e - E}{E} = \pi_{US} - \pi_f,$$

where $\pi_{US} = (P_{US}^e - P_{US})/P_{US}$.[2] Combining this with UIP, we get:

$$\text{(PPP and UIP)}: R_\$ = R_f + \pi_{US} - \pi_f.$$

[2]A simple principle was applied here. Let $\%\Delta x$ denote the percent change in x, *i.e.*,

This relationship shows that our framework implies the *Fisher effect*: a rise in π_{US} translates one-for-one into a rise in $R_{\$}$, assuming the foreign variables, R_f and π_f, do not change. There is a simple intuition for this. PPP implies that if π_{US} is higher, then the rate of depreciation on US currency is greater. But, other things the same, this reduces the rate of return on US financial assets, by comparison with the rate of return on foreign financial assets. In order for people to be happy holding both types of assets, the US interest rate must be higher. It must be higher by exactly the amount of the increased depreciation on the dollar.

Rearranging the previous equation, we obtain:

$$R_{\$} - \pi_{US} = R_f - \pi_f,$$

so that the real rate of interest in the US and other countries must be the same, in the long run.

The money market equation helps us to determine the US inflation rate. In particular,

$$\pi_{US} = \%\Delta M,$$

since $L(R_{\$}, Y)$ is constant under our assumptions. According to this expression, if a 5% inflation is desired in the long run, then to achieve that target money growth must be 5% too.

It is instructive to temporarily drop the assumption that Y is constant. Also, write the money demand equation as $L(R_{\$}, Y) = f(R_{\$})Y^{\gamma}$. Then it is easy to confirm:

$$\pi_{US} = \%\Delta M - \gamma\%\Delta Y.$$

That is, to know what sort of money growth is required to hit a given long-run inflation target, one must have an idea about the economy's long run growth rate. In addition, one must know the value of the elasticity of demand for money with respect to income, γ.[3]

$100(x^e - x)/x$. Then, $xy/z = q$ implies, approximately, that $\%\Delta x + \%\Delta y - \%\Delta z = \%\Delta q$. In the discussion in the text, $x = E$, $y = P_f$, $z = P_{US}$, and $q = 1$.

[3]The parameter, γ, is the elasticity of demand for money for the following reason. Suppose income changes by $\%\Delta Y$, and the other variables that affect the demand for money, P and R, do not change. The resulting percent change in the demand for money is $\%\Delta M = \gamma\%\Delta Y$. Thus,

$$\frac{\%\Delta M}{\%\Delta Y} = \gamma.$$

That is, for every one percent change in Y, the percent change in the demand for money is γ.

3. Experiments.

   (a) One time, permanent increase (jump) in M. The variables to be determined are, P_{US}, $R_{\$}$, E. Conjecture that $R_{\$}$ does not change (in a moment, this conjecture will be verified.) Given this conjecture, the demand for money - L - does not respond to the jump in M, so that for the money demand equation to be satisfied it is necessary that P_{US} jumps by the same percent as the jump in M, so that M/P_{US} remains unchanged. Given the jump in P_{US}, PPP indicates that E and E^e must jump equiproportionally to M too. Suppose the jump in M was x percent, so that the new M and P_{US} are $(1+x)M$ and $(1+x)P_{US}$, respectively. Also, the new E^e and E are, respectively, $(1+x)E^e$ and $(1+x)E$. With this change in the exchange rate, its rate of change does not change in this experiment (verify this by substituting the new E^e and E into the rate of change formula). As a result, the UIP relation can continue to be satisfied at the old $R_{\$}$. This verifies our conjecture that $R_{\$}$ does not change.

   (b) Increase in money growth. Suppose an unexpected change in the rate of money growth occurs in period t_0. The money stock follows the path in the curve in Figure 15-1 (a), on page 378 of KO. Its growth rate is assumed to be some (unspecified) number π before the change. At date t_0, its growth rate becomes $\pi + \Delta\pi$, where $\Delta\pi$ is the notation used to designate the change in the money growth rate.[4] It is important to understand the nature of this experiment, which is very different from the one just discussed, where the money stock took a permanent *jump* at t_0. Here, the value of the money stock does not suddenly change at any point in time (see Figure (a) again). For example, the event at t_0 is not that M jumps, only that its growth rate changes. Our objective now is to figure out the impact of this change on the three variables: P_{US}, $R_{\$}$, E. We also want to know how their growth rates are affected.

      i. Inflation jumps from π_{US} to $\pi_{US} + \Delta\pi$. Why? We know (this will be confirmed momentarily) that whatever happens to $R_{\$}$ in the instant, t_0, it is constant from then on. This means that money demand is constant after instant t_0. But, if money demand is constant, then the ratio, M/P_{US} must be constant too. This means that, after t_0, P_{US} must be growing at the same rate as the new growth rate of M.

[4] Example: is π is .08 and $\Delta\pi$ is .01, then the money growth rate goes from 8 percent to 9 percent.

ii. The interest rate, $R_\$$, jumps at t_0 because of the Fisher effect.

iii. Real money. This drops at t_0 because of the rise in $R_\$$.

iv. The price level. We just showed that the growth rate of P_{US} (i.e., the inflation rate) jumps at t_0. But, what does the price *level* do? Does it jump, or does it behave more like the money stock itself, which was assumed not to jump at t_0? The answer is that P_{US} must jump at t_0. This is the only way that M/P_{US} can drop, given that M does not drop. This explains the price path depicted in Figure 15-1 (c) on page 378.

v. The effect of all this on the exchange rate can be determined from PPP. First, since P_{US} jumps at t_0, then E must too, in the same proportion. Second, since the growth rate of P_{US} jumps by $\Delta\pi$, the growth rate of E must jump by the same amount, according to PPP.

In sum, the increased money growth induces an equal increase in inflation and in the rate of depreciation of the currency. It also induces an immediate jump in the price level and immediate depreciation of the currency.

2 More Sophisticated Model of Exchange Rate (Long Run)

1. Some Facts About Exchange Rates. An equation at the core of the simple monetary approach to exchange rates is PPP. Figures displayed in an earlier lecture - and reproduced here - show that PPP does not work well in practice. There has been a persistent real depreciation of the US currency against the currencies of all six of the countries in the Figures, with the exception of Canada. It just doesn't make sense to think that the real exchange rate always has a tendency to return to unity (or, any other constant) after it is hit by a shock, as PPP supposes.

   It deserves emphasis that PPP pertains to the *real exchange rate*, EP_f/P_{US}. Be sure to distinguish this from the *nominal exchange rate*, E. One corresponds to the relative price of two countries' goods and the other refers to the relative price of the two countries' currencies.

   The difference between the real and nominal exchange rates can best be seen by looking at the data for Italy, the UK and France. The US dollar enjoyed a persistent *appreciation* against these currencies in the period since World War II. Despite this, the real value of the dollar fell against these currencies over the same period. The real exchange rate depreciated. Put differently, the dollar cost of the goods in the Italian, UK, and French consumer baskets rose relative to the cost of goods in the US consumer basket. This is true, even though the dollar cost of those currencies rose over the same period (i.e., the dollar appreciated).

2. A More Sophisticated Model of Exchange Rates.

   Motivated by the previous observations, we replace PPP by a better model of the real exchange rate, one that is not inconsistent with the trends observed in real exchange rates. The new model is only a little more complicated. We don't want our modified model to capture *all* of the reasons (i.e., differences in monopoly power, trade restrictions, differences in baskets, etc.) that real exchange rates vary. Such a model would be too huge to be workable. What we need instead, is a model that captures the essence of what drives the real exchange rate around, without getting too involved in details.

   We will think of there being two main forces operating on real exchange rates: demand and supply.

   (a) Demand. When demand in the world (i.e., by foreigners and/or Americans) shifts towards US goods, then we expect the real exchange rate to fall (i.e., a real appreciation of the US dollar). That is, we expect the shift in demand away from foreign goods to reduce their price, EP_f, and raise the price of American goods.

   This simple idea encompasses various possibilities:

   i. Suppose there are traded goods and nontraded goods, and the law of one price applies to the traded goods. Suppose Americans increase their demand for American produced nontradeables. Since, by definition nontraded goods are only produced in the US, the rise in demand for them by Americans is likely to press hard on US productive resources. This is likely to raise the price of nontradeables relative to tradeables: P_{US}^{NT}/P_{US}^{T}. Then,

$$
\begin{aligned}
\frac{EP_{foreign}}{P_{US}} &= E\frac{a_1 P_f^T + a_2 P_f^{NT}}{b_1 P_{US}^T + b_2 P_{US}^{NT}} \\
&= E\frac{a_1\left(P_f^T/P_{US}^T\right) + a_2\left(P_f^{NT}/P_f^T\right)\left(P_f^T/P_{US}^T\right)}{b_1 + b_2 P_{US}^{NT}/P_{US}^T} \\
&= \frac{a_1 + a_2\left(P_f^{NT}/P_f^T\right)}{b_1 + b_2\left(P_{US}^{NT}/P_{US}^T\right)},
\end{aligned}
$$

since $EP_f^T/P_{US}^T = 1$ by the law of one price. From this expression, it is clear that the rise in American demand for US nontradeables will produce a fall in the US real exchange rate.

ii. Suppose the world only has traded goods. Americans make oranges and foreigners make apples. Americans primarily consume oranges, but they also consume a few apples. For foreigners it is the reverse: they primarily consume apples, but they also consume a few oranges. Now suppose the world wants to eat more oranges and fewer apples (this could be because Americans' preferences have shifted, or foreigners' preferences have shifted). Then, we'd expect the dollar price of apples to fall relative to the dollar price of oranges. This is just the real exchange rate.

(b) Supply. Suppose Americans become more efficient at making whatever they make (say, oranges). Then, we'd expect the price of these to fall as their supply rises. This will produce a rise in the real exchange rate (the price of apples relative to oranges), or a real depreciation of the dollar.

Our modified model is composed of the money market equation, UIP and a modified version of PPP:

$$\begin{aligned}
\text{UIP} &: \quad R_\$ = R_f + \frac{E^e - E}{E}. \\
\text{Money Market} &: \quad \frac{M}{P_{US}} = L(R_\$, Y). \\
\text{Real Exchange Rate Determination} &: \quad q = f(\text{demand}, \text{supply}). \\
\text{Real Exchange Rate} &: \quad \frac{EP_f}{P_{US}} = q,
\end{aligned}$$

The first three equations are 'behavioral equations', they summarize the assumptions we have made about the way market participants make decisions. The last equation is just the definition of q, the real exchange rate.

3. Analysis Using the More Sophisticated Model. In this model, a change in the money stock or its growth rate has the same effect as in the Monetary Approach. That is because we assume monetary factors don't (in the long run) affect the demand and supply conditions which impact on q. The novelty of this framework is that it can be used to study the impact on E, P_{US} and $R_\$$ of a change in q.

   (a) Effects of a Change in Demand for American Goods. Consider the effect of an increase in world demand for American goods.

Suppose it induces a one-time, permanent drop in q, i.e., induces a real appreciation of the dollar. There is no change in the growth rate in q. Now, suppose $R_\$$ does not change (we will verify this assumption in a moment). Then, the money market condition says P_{US} does not change either, since the other variables in that relation, M, Y, do not change by assumption (M is determined by the Fed, while Y is determined by the amount of capital and people, etc. in the country). If P_{US} does not change then the real exchange rate relation indicates that E has to drop in proportion to the change in q. That is, E appreciates instantly. But, since there is no change in the growth rate of q or P_{US}, there is no change in $(E^e - E)/E$ either. UIP then implies that $R_\$$ does not change, verifying our assumption to this effect, made above.

The analysis of a change in the supply which affects q is the same.

(b) Other Implications of the More Sophisticated Model.

i. International Interest Rate Differentials. The real exchange rate expression has the following growth rate implication:

$$\frac{E^e - E}{E} = \frac{q^e - q}{q} + \pi_{US} - \pi_f.$$

That is, the rate of depreciation in the nominal exchange rate is the sum of the depreciation in the real exchange rate, plus the excess of US inflation over that of the foreign country. Under PPP, real exchange rate depreciation is ruled out. However, the data force us to bring it in. Obviously, the data are characterized by long-term, persistent movements in q. If we substitute this into the interest parity relation, we obtain:

$$R_\$ - R_f = \frac{q^e - q}{q} + \pi_{US} - \pi_f.$$

So, interest rate differentials reflect not just inflation differentials, but also the trend change in the real exchange rate. The Fisher effect continues to hold, as long as the factor increasing π_{US} does not affect $(q^e - q)/q$ (or R_f, π_f, but we already had to assume that before). In this case, a jump in π_{US} shows up one-for-one in the form of a jump in $R_\$$.

ii. There is a different way to write the previous expression for international nominal interest rate differentials. Note that $R_\$ - \pi_{US}$ is the real interest rate in the US and $R_f - \pi_f$ is the foreign real interest rate.[5] Then, rewriting the last equation, you get:

$$r_{US}^e - r_f^e = \frac{q^e - q}{q},$$

where $r_{US}^e = R_\$ - \pi_{US}$ is the real interest rate in the US. Thus, the real interest rate differential between two countries is zero if PPP holds (in which case $q^e = q = 1$), or non-zero if q is expected to change.

3 Analysis of the Short and Long Run

1. Back to the Short Run: Making Y Endogenous.

   Up to now, we have assumed that Y is exogenous in the short and the long run. We will continue to maintain this assumption for the long run. We suppose that in the long run output is determined by the size of the population, the level of education, the amount of physical capital, etc. However, we now drop the assumption that Y is exogenous in the short run. We do this because there are reasons to think that Y reacts in the short run to changes in the exogenous variables. For example, the recession in the early 1980s, a transitory slowdown in Y, is often attributed to a tight money policy adopted at the time by the Fed.

   (a) Where we stand. Short run: endogenous variables are R and E (when there is no subscript on a variable like R, we can assume that it applies to the domestic economy, or to the US.) We have two relationships to pin these down - the money market condition and UIP. In the long run the endogenous variables are P, R and E. We have three relationships to pin these down - the money market condition, UIP and the condition that the real exchange rate, q is determined (rather vaguely) by exogenous 'demand' and 'supply'. So far, output, Y, has been held fixed at its long-run level, which

[5] Remember what a real interest rate is. It's the ratio of the goods value of what you earn on an asset, to the goods value of what it costs. Consider a US asset with a nominal return of $1 + R_\$$. The cost of one unit of this asset is one US dollar, which corresonds to $1/P_{US}$ goods. Later, you get back $1 + R_\$$ dollars, which translates into $(1 + R_\$)/P_{US}^e$ goods, where P_{US}^e is the expected price level. Thus, the real rate of return is

$$\frac{(1 + R_\$)/P_{US}^e}{1/P_{US}} = \frac{1 + R_\$}{1 + \pi_{US}} \simeq 1 + R_\$ - \pi_{US},$$

where, $\pi_{US} = (P_{US}^e - P_{US})/P_{US}$.

is determined by the amount of people, capital, education, etc., in the economy.

We now want to add Y to the list of our model's variables that are endogenous in the short run. We will stick to our previous assumption that, in the long run, Y is determined by factors that are unrelated to the other exogenous variables in our model. To make Y endogenous in the short run, however, we must add one more relationship, beside just the money market condition and UIP, to our short run model. We will add a goods market relationship. This same relationship will also be used to make precise the relationship we have been using in our long-run model, which relates q to demand and supply.

(b) Aggregate Demand and Making Output Endogenous in the Short Run. Aggregate demand is the sum of desired (or, planned) spending by households on consumption goods, by business on investment goods, by government, plus net exports:

$$\text{Aggregate Demand: } D = C(Y-T) + I + G + CA(\frac{EP^*}{P}, Y-T).$$

Here, planned household consumption, C, is an increasing function of disposable income, $Y-T$,[6] I denotes planned investment and G denotes planned government spending. Also, the planned current account (primarily, exports minus imports), CA, is increasing in its first argument and decreasing in its second.[7] (Sorry for a switch in notation here. Here, P^* is the foreign price level, something we have previously denoted by P_f.) Equilibrium in the goods market corresponds to a situation where planned spending equals output, $Y = D$.

Be careful to distinguish the goods market equilibrium condition from the national income identity, which must hold whether or not the goods market is in equilibrium. The national income identity says that total output must be equal to *actual* consumption, plus actual investment, plus actual government spending, plus the actual current account. We will assume actual and planned coincide for all components of the national income identity, except investment. So, the goods market is in equilibrium if, and only if,

[6] $C(Y-T)$ denotes the consumption function. This is not a number, C, times $Y-T$. It is a function saying how much people with disposable income, $Y-T$, consume.

[7] Again, $CA(\frac{EP^*}{P}, Y-T)$ is a function. The first argument is the real exchange rate and the second is disposable income. We follow the book in referring to the current account and net exports as synonymous. This is not quite right, since the current account also includes net investment income flows.

planned and actual investment are equal. When the goods market is out of equilibrium, a part of actual investment is unplanned. For example, when aggregate demand is less than output, then actual investment exceeds planned investment. The excess of actual over planned is assumed to be composed of an unplanned accumulation of inventories. This makes sense. When demand is low, we'd expect to see goods pile up on store shelves. In the national income accounts this is counted as the inventory accumulation part of investment by firms (this part of investment by firms is unplanned and undesired). When aggregate demand is high, then unplanned investment is negative: inventories are disappearing from store shelves.

Because these observations are so important for the analysis that follows, we dwel on them a little longer. Let C^p, I^p, G^p, CA^p denote planned consumption, investment, government spending and the current account, respectively. Here, $C^p = C(Y-T)$, and $CA^p = CA(q, Y-T)$, where q is the real exchange rate. Let C^a, I^a, G^a, CA^a denote the amount of consumption, investment, government spending, current account that actually occurs. The following is an accounting identity and is always true:

$$Y = C^a + I^a + G^a + CA^a.$$

This is always true, because one way to measure Y is to add up the terms on the right side of the equality. But, $Y = C^p + I^p + G^p + CA^p$ is *only* true when the goods market is in equilibrium. Now, we assume that households always get what they plan, so that $C^p = C^a$. Similarly for government and the current account. We do not assume that business always does the amount of investment they plan. For example, suppose planned spending is less than $Y : Y > C^p + I^p + G^p + CA^p$. We assume that this excess of output over planned spending shows up as unintended inventory accumulation, I^u. That is,

$$I^u = Y - \left(C^p + I^p + G^p + CA^p\right).$$

Then,

$$I^a = I^u + I^p.$$

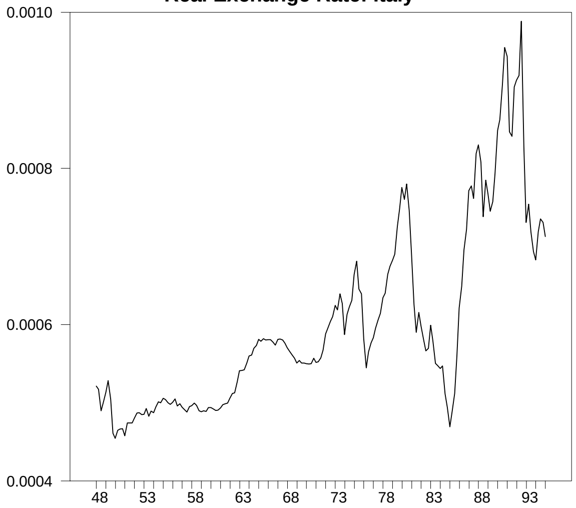
It is easy to verify that with this definition of I^a and I^u, the national income identity is satisfied.

Does all this make sense? Of course! For households to realize their plans they just have to go to the store, or restaurant and wilfully make the purchase. The decision is up to no one but the household itself. The same is true for the government and for the export and import decisions that go into determining the current account. This is why it makes sense to posit $C^a = C^p$, $G^a = G^p$ and $CA^a = CA^p$. But, things are different for firms' investment decisions. Certainly, to purchase a new piece of machinery the firm just has to go out and do it. But, part of investment is inventory investment. This component of firms' investment decision is not entirely under the control of firms. If households stop buying things, goods pile up unsold on store shelves. This pile up of goods is counted in the inventory investment of firms. It is included in inventory investment whether firms planned on it or not. It is because the level of inventory investment is not determined by the decisions of firms alone, that it makes sense to think of the possibility that I^a is not equal to I^p.

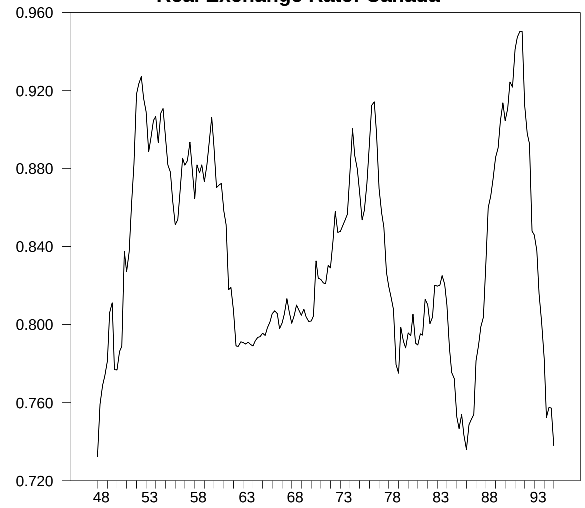
From here on, we will not use the superscripts, 'a' and 'p'. This should not cause confusion. The distinction only matters for investment. Which superscript we have in mind should be clear from the context.

The goods market equilibrium condition will be used in both our long run and short run analysis. In the short run, when $Y > D$, so that there is unplanned inventory accumulation, Y falls. In the long run, when Y is exogenous, we will suppose that $Y > D$ leads to a fall in the relative price of domestic goods, i.e., to a rise in q.

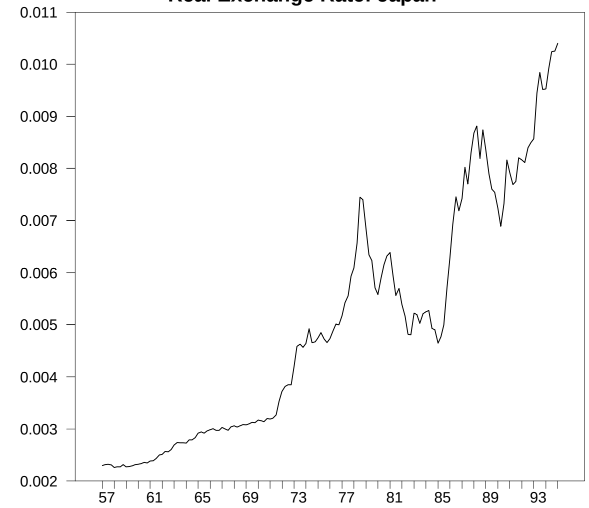
Real Exchange Rate: Italy



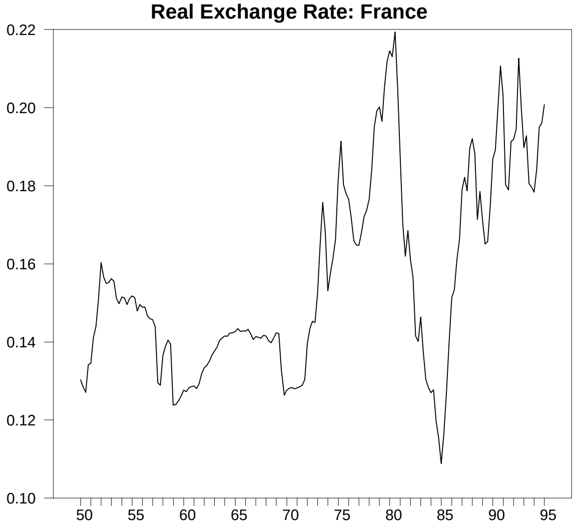
Real Exchange Rate: Canada



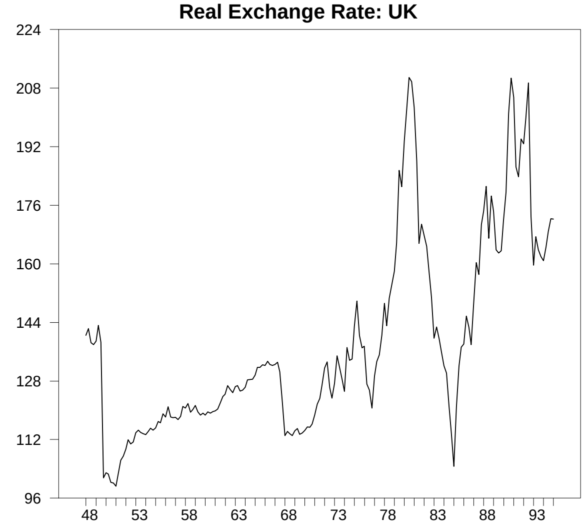
Real Exchange Rate: Japan



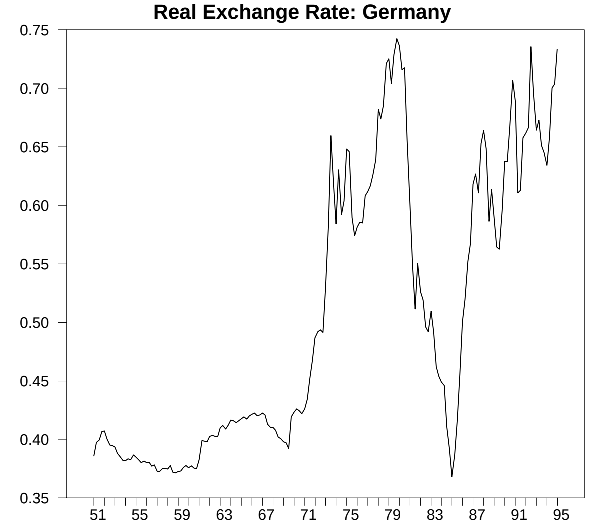
Real Exchange Rate: France



Real Exchange Rate: UK

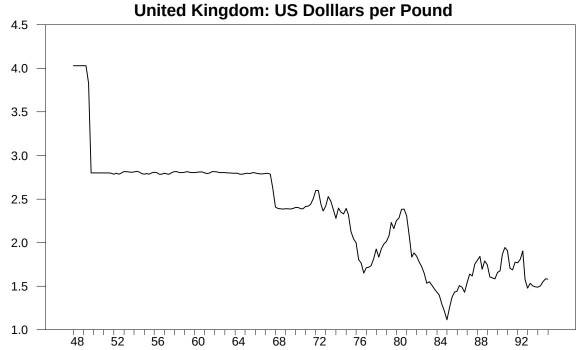


Real Exchange Rate: Germany

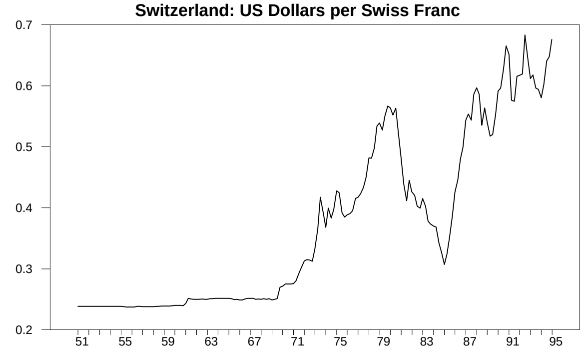


Italy: US Dollars per Lira

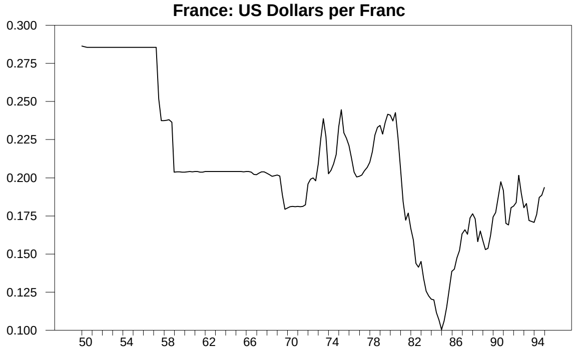
United Kingdom: US Dolllars per Pound



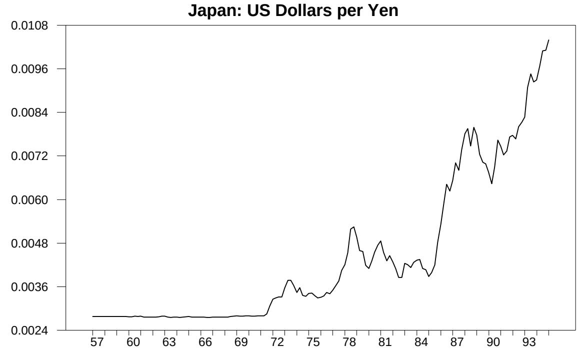
Switzerland: US Dollars per Swiss Franc



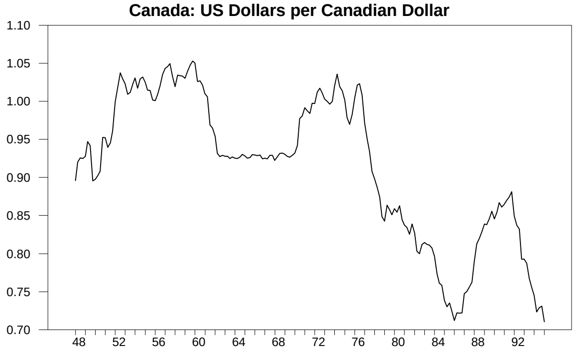
France: US Dollars per Franc



Japan: US Dollars per Yen



Canada: US Dollars per Canadian Dollar



Germany: US Dollars per Mark