The Effects of Balance Sheet Constraints on Non Financial Firms

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Background

• Several shortcomings of standard New Keynesian model.
  – It assumes that the interest rate satisfies an Euler equation with the consumption of a single, representative household.
  – Evidence against that Euler equation is strong (Hall (JPE1978), Hansen-Singleton (ECMA1982), Canzoneri-Cumby-Diba (JME2007))

• Here, discuss Buera-Moll (AEJ-Macro2015) model of heterogeneous households and firms.
  – Shows how a model with heterogeneous households breaks Euler equation.
  – Shows how deleveraging can lead to many of the things observed in the Financial Crisis and Great Recession.
    • fall in output, investment, consumption, TFP, real interest rate.

• ‘Toy’ model that can be solved analytically, great for intuition.

• Earlier, similar models: Kahn-Thomas (JPE2013), Liu-Wang-Zha (ECMA2013).
Outline

• Hand-to-mouth workers

• Entrepreneurs (where all the action is)

• Aggregates: Loan Market, GDP, TFP, Consumption, Capital, Consumption

• Equilibrium
  – Computation.
  – Parameter values.
  – The dynamic effects of deleveraging.
Hand-to-mouth Workers

• Hand-to-mouth workers maximize

\[ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t^W)^{1-\sigma}}{1 - \sigma} - \frac{1}{1 + \chi} L_t^{1+\chi} \right] \]

subject to:

\[ C_t^W \leq w_t L_t. \]

• Solution:

\[ L_t^{\frac{\chi+\sigma}{1-\sigma}} = w_t, \quad (1) \]

and labor supply is upward-sloping for \( 0 < \sigma < 1 \).
Entrepreneurs

• $i^{th}$ entrepreneur would like to maximize utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}), \ u(c) = \log c.$$  

• $i^{th}$ entrepreneur can do one of two things in $t$:
  
  – use time $t$ resources plus debt, $d_{i,t} \geq 0$, to invest in capital and run a production technology in period $t + 1$.
    • will do this if $i$’s technology is sufficiently productive.
  
  – use time $t$ resources to make loans, $d_{i,t} < 0$, to financial markets.
    • will do this if $i$’s technology is unproductive.
Rate of Return on Entrepreneurial Investment

• $i^{th}$ entrepreneur can invest $x_{i,t}$ and increase its capital in $t + 1$:

$$k_{i,t+1} = (1 - \delta) k_{i,t} + x_{i,t}, \quad \delta \in (0, 1)$$

• In $t + 1$ entrepreneur can use $k_{i,t+1}$ to produce output:

$$y_{i,t+1} = (z_{i,t+1} k_{i,t+1})^\alpha l_{i,t+1}^{1-\alpha}, \quad \alpha \in (0, 1),$$

where $l_{i,t+1} \sim$ amount of labor hired in $t + 1$ for wage, $w_{t+1}$.

• Technology shock, $z_{i,t+1}$, observed at time $t$, and
  – independent and identically distributed:
    • across $i$ for a given $t$,
    • across $t$ for given $i$.
  – Density of $z$, $\psi(z)$; CDF of $z$, $\Psi(z)$. 
Rate of Return on Entrepreneurial Investment

• $i^{th}$ entrepreneur’s time $t + 1$ profits:

$$\max_{l_{i,t+1}} \left[ (z_{i,t+1}k_{i,t+1})^\alpha l_{i,t+1}^{1-\alpha} - w_{t+1}l_{i,t+1} \right]$$

$$= \pi_{t+1}z_{i,t+1}k_{i,t+1}$$

$$\pi_{t+1} \equiv \alpha \left( \frac{1 - \alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}}.$$

• Rate of return on one unit of investment in $t$:

$$\pi_{t+1}z_{i,t+1} + 1 - \delta.$$
The Decision to Invest or Lend

- The $i^{th}$ entrepreneur can make a one period loan at $t$, and earn $1 + r_{t+1}$ at $t+1$.

- Let $\bar{z}_{t+1}$ denote value of $z_{i,t+1}$ such that return on investment same as return on making a loan:
  
  \[ \pi_{t+1}\bar{z}_{t+1} + 1 - \delta = 1 + r_{t+1}. \]

- If $z_{i,t+1} > \bar{z}_{t+1}$,
  - borrow as much as possible, subject to collateral constraint:
    \[ d_{i,t+1} \leq \theta_t k_{i,t+1}, \quad \theta_t \in [0,1], \]
    and invest as much as possible in capital.
  - In this case borrow:
    \[ d_{i,t+1} = \theta_t k_{i,t+1}. \]

- If $z_{i,t+1} < \bar{z}_{t+1}$, then set $k_{i,t+1} = 0$ and make loans, $d_{i,t} < 0$.
Entrepreneur’s Problem

• At $t$, maximize utility,

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{i,t+j})$$

subject to:

- given $k_{i,t}$ and $d_{i,t}$
- borrowing constraint
- budget constraint:

$$c_{i,t} + k_{i,t+1} - (1 - \delta) k_{i,t} \leq y_{i,t} - w_{i,t}, \text{ if entrepreneur invested in } t - 1$$
$$\pi_t z_{i,t} k_{i,t} + d_{i,t+1} - (1 + r_t) d_{i,t}$$

increase in debt, net of financial obligations

• Alternative representation of budget constraint:

$$\equiv k_{i,t+1} - d_{i,t+1}, \text{ ‘net worth’}$$
$$c_{i,t} + a_{i,t+1} \leq [\pi_t z_{i,t} + 1 - \delta] k_{i,t} - (1 + r_t) d_{i,t} \equiv m_{i,t}, \text{ ‘cash on hand’}$$
Entrepreneur’s Problem

• At \( t \), maximize utility,

\[
E_t \sum_{j=0}^{\infty} \beta^j u(c_{i,t+j}),
\]

\( u(c) = \log(c) \), subject to:
- given \( k_{i,t} \) and \( d_{i,t} \)
- borrowing constraint
- budget constraint:

\[
c_{i,t} + a_{i,t+1} \leq m_{i,t}
\]

where,

\[
a_{i,t+1} = k_{i,t+1} - d_{i,t+1}, \quad m_{i,t} = [\pi_t z_{i,t} + 1 - \delta] k_{i,t} - (1 + r_t) d_{i,t}
\]

• Optimal choice of next period’s net worth:

\[
a_{i,t+1} = \beta m_{i,t}, \quad c_{i,t} = (1 - \beta) m_{i,t}.
\]
Entrepreneur’s Problem

- For $z_{i,t+1} \geq \bar{z}_{t+1}$, max debt and capital:

\[ d_{i,t+1} = \theta_t k_{i,t+1} = \theta_t (d_{i,t+1} + a_{i,t+1}) \]

\[ \rightarrow d_{i,t+1} = \frac{\theta_t}{1 - \theta_t} a_{i,t+1}, \quad k_{i,t+1} = \frac{1}{1 - \theta_t} a_{i,t+1} \]

- Example:
  - if $\theta_t = \frac{2}{3}$, then leverage $= 1/ (1 - \theta_t) = 3$.
  - if net worth, $a_{i,t+1} = 100$, then $k_{i,t+1} = 300$ and $d_{i,t+1} = 200$.

- For $z_{i,t+1} < \bar{z}_{t+1}$, $k_{i,t+1} = 0$ and $d_{i,t+1} < 0$ (i.e., lend)
  - upper bound on lending: $d_{i,t+1} = -m_{i,t}$, all cash on hand.
  - won’t go to upper bound with log utility.
Aggregates: Demand for Loans

• The total amount of cash on hand for all entrepreneurs, $M_t$, is

$$M_t = \int m_{i,t} di.$$  

• Total demand for loans:
  
  – Since the $z_{i,t+1}$’s are distributed randomly to entrepreneurs, the cash in hand of the $[1 - \Psi (\bar{z}_{t+1})]$ investing entrepreneurs is:

$$[1 - \Psi (\bar{z}_{t+1})] M_t.$$  

  – Each of these entrepreneurs borrows $d_{i,t+1} = \theta_t / (1 - \theta_t) \beta m_{i,t}$, so total borrowing by investing entrepreneurs is

$$\beta \frac{\theta_t}{1 - \theta_t} [1 - \Psi (\bar{z}_{t+1})] M_t.$$
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  - Each of these entrepreneurs borrows $d_{i,t+1} = \theta_t / (1 - \theta_t) \beta m_{it}$, so total borrowing by investing entrepreneurs is

$$\beta \frac{\theta_t}{1 - \theta_t} [1 - \Psi(\bar{z}_{t+1})] M_t.$$
Aggregates: Supply of Loans and Loan Market Clearing

• Total supply of loans:
  – Since the $z_{i,t+1}$’s are distributed randomly to entrepreneurs, the cash in hand of the $\Psi(\bar{z}_{t+1})$ non-investing entrepreneurs is:
    \[
    \Psi(\bar{z}_{t+1})M_t.
    \]
  – Each of these entrepreneurs lends $-d_{i,t+1} = \beta m_{it}$, so total borrowing by investing entrepreneurs is
    \[
    \beta \Psi(\bar{z}_{t+1})M_t.
    \]

• Loan market clearing implies:
  \[
  \beta \frac{\theta_t}{1 - \theta_t} \left[1 - \Psi(\bar{z}_{t+1})\right]M_t = \beta \Psi(\bar{z}_{t+1})M_t,
  \]
  or,
  \[
  \Psi(\bar{z}_{t+1}) = \theta_t. \quad (2)
  \]
Aggregates: Gross Domestic Product

- The $i^{th}$ firm’s production function is:

$$y_{it} = (z_{i,t}k_{i,t})^\alpha l_{i,t}^{1-\alpha} = \left(\frac{z_{i,t}k_{i,t}}{l_{i,t}}\right)^\alpha l_{i,t}. $$

- Ratios equal ratio of sums:

$$\frac{z_{i,t}k_{i,t}}{l_{i,t}} = \frac{\int_i z_{i,t}k_{i,t}di}{\int_i l_{i,t}di} = \frac{\int_i z_{i,t}k_{i,t}di}{L_t}. $$

- GDP

$$Y_t = \int_i y_{i,t}di = \left(\frac{\int_i z_{i,t}k_{i,t}di}{L_t}\right)^\alpha \int_i l_{i,t}di$$

$$= \left(\frac{\int_i z_{i,t}k_{i,t}di}{L_t}\right)^\alpha L_t$$
Aggregates: GDP, TFP and wage

• With some algebra, can establish:

\[ Y_t = \left( \int_i z_{i,t} k_{i,t} di \right)^\alpha = E[z|z>\bar{z}_t] \times K_t \]

\[ = (E[z|z>\bar{z}_t])^\alpha \times K_t \]

\[ L_t^{1-\alpha} = Z_t K_t L_t^{1-\alpha}, \quad (3) \]

\[ Z_t \equiv (E[z|z>\bar{z}_t])^\alpha. \quad (4) \]

• Simple intuition:
  – Aggregate output, \( Y_t \), a function of aggregate capital and labor, and (endogenous) TFP, \( Z_t \).
  – \( Z_t \) average TFP of firms in operation.

• Aggregate wage:

\[ w_t = (1-\alpha) \frac{Y_t}{L_t}. \quad (5) \]
Aggregates: Consumption

- Integrating over entrepreneurs’ budget constraints:

\[ \int_i [c_{i,t} + k_{i,t+1} - d_{i,t+1}] di \]

\[ = \int_i [y_{i,t} - w_t l_{i,t} + (1 - \delta) k_{i,t} - (1 + r_t) d_{i,t}] di \]

- Using loan market clearing, \( \int_i d_{i,t} di = 0 \):

\[ \begin{aligned}
&C_t^E + K_{t+1} - (1 - \delta) K_t = Y_t - (1 - \alpha) Y_t, \\
where
&C_t^E = \int_i c_{i,t} di
\end{aligned} \]
Aggregates: Capital Accumulation

• Entrepreneur decision rule:

\[ a_{i,t+1} \equiv k_{i,t+1} - d_{i,t+1} \]
\[ = \beta [y_{i,t} - \omega_t l_{i,t} + (1 - \delta) k_{i,t} - (1 + r_t) d_{i,t}] \]

• Integrating over all entrepreneurs (using \( \int_i d_{i,t} di = 0 \)):

\[ K_{t+1} = \beta [\alpha Y_t + (1 - \delta) K_t] \]  \( (7) \)

• Note: \( K_{t+1} \) is not a direct function of \( \theta_t \).
  – If \( \theta_t \) falls, then borrowing drops by investing entrepreneurs, driving down \( r_{t+1} \).
  – Lower \( r_{t+1} \) encourages unproductive entrepreneurs who previously were lending, to switch to borrowing and buying more capital.
  – The positive and negative effects on capital purchases cancel, which is why \( K_{t+1} \) is not a function of \( \theta_t \).
Interestingly, aggregate entrepreneurial consumption satisfies Euler equation:

\[
\frac{C_{t+1}^{E}}{C_{t}^{E}} = \frac{(1 - \beta) \left[ \alpha Y_{t+1} + (1 - \delta) K_{t+1} \right]}{(1 - \beta) \left[ \alpha Y_{t} + (1 - \delta) K_{t} \right]}
\]

\[
= \beta \frac{\alpha Y_{t+1} + (1 - \delta) K_{t+1}}{K_{t+1}}
\]

\[
= \beta \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right]
\]

But, does not hold for aggregate consumption, \( C_{t} = C_{t}^{W} + C_{t}^{E} \). does not hold relative to the interest rate.
Equilibrium

• Seven variables:

\[ L_t, w_t, C_t^E, Y_t, K_{t+1}, \bar{z}_t, Z_t. \]

• Seven equations: (1), (2), (3), (4), (5), (6), (7).

• Exogenous variables:

\[ K_1, \theta_0, \theta_1, \theta_2, \ldots, \theta_T \]
Equilibrium Computation

- Responses to exogenous variables:
  - For $t = 1, 2, ..., T$, $\bar{z}_t = \Psi^{-1}(\theta_{t-1})$ using (2);
    $Z_t \equiv (E[z|z > \bar{z}_t])^{\alpha}$ using (4),
  - Using (1) and (5) for $L_t$ and $\omega_t$; (3) for $Y_t$; (7) for $K_{t+1}$; and (6) for $C_t^E$:

$$L_t = \left((1 - \alpha)Z_tK_t^\alpha\right)^{\frac{1-\sigma}{\chi+\sigma+(1-\sigma)\alpha}}$$

$$\omega_t = L_t^{\frac{\chi+\sigma}{1-\sigma}}$$

$$Y_t = Z_tK_t^\alpha L_t^{1-\alpha}$$

$$K_{t+1} = \beta [\alpha Y_t + (1 - \delta) K_t]$$

$$C_t^E = (1 - \beta) [\alpha Y_t + (1 - \delta) K_t]$$

sequentially, for $t = 1, 2, 3, ..., T$. 
Equilibrium Computation

• Other variables: interest rate and profits for \( t = 1, 2, \ldots, T \):

\[
\pi_t = \alpha \left( \frac{1 - \alpha}{\omega_t} \right)^{\frac{1-\alpha}{\alpha}}
\]

\[
1 + r_t = \pi_t \bar{z}_t + 1 - \delta
\]

• Pareto distribution:

\[
\psi(z) = \eta z^{-(\eta + 1)}, \; \eta = 2.1739, \; 1 \leq z
\]

\[
\Psi(\bar{z}) = 1 - \bar{z}^{-\eta}, \; Ez = \frac{\eta}{\eta - 1} = 1.85.
\]
Parameter Values and Steady State

- Other parameters:
  \[ \alpha = 0.36, \delta = 0.10, \beta = 0.97, \chi = 1, \sigma = 0.9. \]

- Steady state, with \( \theta = \frac{2}{3} \):
  \[ Y = 3.45, K = 9.50, L = 1.04, C = 2.50, \]
  \[ w = 2.12, \bar{z} = 1.66, Z = 1.50, \]
  \[ Z^\frac{1}{\bar{x}} = E[z|z > \bar{z}] = 3.07, 1 + r = 0.97, \]
  \[ C^E / C = 0.12, C^W / C = 0.88, \]

\( () \) after rounding.
Tighter Lending Standards: $\theta_t$ down

- ‘MIT shock’
  - economy in a steady state, $t = -\infty, ..., 1, 2$, and expected to remain there.
  - In $t = 3$, $\theta_3$ drops unexpectedly from 0.67 to 0.60, and gradually returns to its steady state level:
    - $\theta_3 = \theta \times 0.9, \theta_t = (1 - \rho) \theta + \rho \theta_{t-1},$ for $t = 4, 5, ...$
    - $\rho = 0.8.$
Immediate Impact of Negative $\theta_t$ Shock

- Period $t = 3$ impact of shock:
  - Deleveraging associated with drop in $\theta_3$ reduces demand for debt by each investing entrepreneur, driving down period $t = 3$ interest rate, $r_4$.
  - Marginally productive firms which previously were lending, switch to borrowing and making low-return investments with the drop in $r_4$.

- No impact on total investment in period $t = 3$, as the cut-back by high productivity entrepreneurs is replaced by expanded investment by lower productivity entrepreneurs.
- No impact on consumption, wages, etc., in period $t = 3$. 
Immediate Impact of Negative $\theta_t$ Shock

Lower $r_{t+1}$ implies low productivity. Entrepreneurs switch from lending to borrowing.

Demand $\theta_t$ down

Supply
Dynamic Effects of Drop in $\theta_t$

- The cut in leverage by highly productive, but collateral-poor, firms is the trigger for the over 1.8 percent drop in TFP in period $t = 4$.
  - Until the drop in capital is more substantial, by say period $t = 20$, the drop in TFP is the main factor driving GDP down.

- Total consumption drops substantially, driven by the drop in income of hand-to-mouth workers, who consume $2/3$ of GDP.
  - Entrepreneurial consumption, directly related to GDP, also drops.

- Investment drops by over 2 percent.

- Employment drops by (a modest) 0.1 percent.
Response to Collateral Constraint Shock

- **Capital, $K_t$**
- **Output**
- **Consumption**
- **Wage**
- **Labor supply**
- **Investment**
- **Interest rate, $1 + r_{t+1}$**
- **Leverage**
- **Measured TFP: $Z_t$**
Conclusion

- Buera-Moll model gives a flavor of the sort of analysis one can do with heterogeneous agent models with balance sheet constraints.
  - Illustrates the value of simple models for gaining intuition.

- Model provides an ‘endogenous theory of TFP’.
  - Stems from poor allocation of resources due to frictions in financial market.
  - See also Song-Storesletten-Zilibotti (AER2011).

- Deleveraging shock gets a surprising number of things right, but
  - how important was deleveraging per se, for the crisis?
  - what is the ‘deleveraging shock a stand-in for?’