Advanced Macroeconomics,

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Econ 416

Homework #8

Due: November 28

1. Consider the New Keynesian model in question 4 in the take-home exam. When we mechanically apply the methods of the linearization handout, we end up with a representation of the model variables which is of the 'non-invertible a' form. Simulating the sunspot in this case is complicated and it is convenient to find a way to squeeze the system into the invertible a case. A suggestion for how to do this is described below.[1] The model is expressed as follows:

$$\begin{aligned}\delta E_t\pi_{t+1} + \lambda y_t - \pi_t &= 0 \\ E_t y_{t+1} - y_t - \frac{1}{\sigma}(r_t - E_t\pi_{t+1}) &= 0 \\ \rho r_{t-1} + (1-\rho)\beta E_t\pi_{t+1} + (1-\rho)\gamma y_t - r_t &= 0.\end{aligned} \tag{1}$$

The object, y_t, is the output gap and the natural rate of interest does not appear here because there are no fundamental shocks. Expectations are nevertheless included because, although there are no fundamental shocks, we want to allow for the possibility of stochastic equilibria. Define

$$\begin{aligned}\pi_{t+1} &= E_t\pi_{t+1} + \eta_{t+1} \\ y_{t+1} &= E_t y_{t+1} + \psi_{t+1},\end{aligned}$$

where η_{t+1} and ψ_{t+1} denote the unexpected components in π_{t+1} and y_{t+1}, respectively. These objects must satisfy $E_t\eta_{t+1} = E_t\psi_{t+1} = 0$. Consider the following definition of Y_t :

$$Y_t = \begin{pmatrix} \pi_t \\ y_t \\ r_{t-1} \end{pmatrix}.$$

Note that the last term in Y_t is r_{t-1}, a variable that is predetermined at time t. Suppose the system starts in period 0, so that there are two free variables in Y_0. Find the matrices a, b and the shocks vector, ω_t, in the first order VAR representation for Y_t. Note that this

[1] The suggestion is taken from Clarida, Gali and Gertler (NBER working paper 6442).

system is in squarely in the 'invertible a' case in the linear solution methods handout, 'Solutions to a Class of Linear Expectational Difference Equations'. With one exception, use the parameter values studied in the section of that handout which works with the NK model. The exception is the value of the coefficient, β, on inflation in the Taylor rule.

(a) Set $\beta = 0.5$ and report the matrix, Π, and its eigenvalues.

(b) When $\beta = 0.5$, is the steady state of the model determinate, or indeterminate? Explain carefully, being sure to be clear about the meaning of the phrases, *determinacy of steady state equilibrium* and *indeterminacy of steady state equilibrium.*

(c) Show that we cannot construct a non-explosive solution driven by two independent sunspots.

(d) Show that there is a non-explosive solution to model in which the variables are driven by a single sunspot shock. Similate a sequence of 500 artificial observations drawn from this equilibrium. Compute the correlation between inflation and output.

(e) Suppose that $\beta = 1.5$. Explain why there is no non-explosive sunspot solution.

2. This question and the next explore a couple of aspects of empirical analysis, using the NK model as a laboratory. We take the version of the model studied in class, in which there is a technology shock and a shock to preferences. For definiteness, here is the model:

$$\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \text{ (Calvo pricing equation)} \\
x_t &= -\left[r_t - E_t \pi_{t+1} - r_t^*\right] + E_t x_{t+1} \text{ (intertemporal equation)} \\
r_t &= \alpha r_{t-1} + (1-\alpha)\left[\phi_\pi \pi_t + \phi_x x_t\right] \text{ (policy rule)} \\
r_t^* &= \rho \Delta a_t + \frac{1}{1+\varphi}(1-\lambda)\tau_t \text{ (natural rate)} \\
y_t^* &= a_t - \frac{1}{1+\varphi}\tau_t \text{ (natural output)} \\
x_t &= y_t - y_t^* \text{ (output gap)} \\
\Delta a_t &= \rho \Delta a_{t-1} + \varepsilon_t^a, \ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau,
\end{aligned}$$

where κ denotes the slope of the Phillips curve. The connection of κ to the Calvo price stickiness parameter, θ, and the Frisch labor supply elasticity parameter, φ, is described in the class handout on the NK model.

Consider the following parameterization:

$$\begin{aligned}\beta &= 0.97,\ \phi_x = 0.15,\ \phi_\pi = 1.5,\ \alpha = 0.8,\ \rho = 0.9,\ \lambda = 0.5,\\ \varphi &= 1,\ \theta = 0.75,\ \sigma_a = \sigma_\tau = 0.02.\end{aligned}$$

(a) Generate $T = 200$ artificial observations on the model variables (if you use Dynare, you can do this by including the argument, periods = 200, in the stoch_simul command). Include, among the variables, the growth rate of actual output (recall, the model is in log linear form). In case you use Dynare, the simulated variables are placed in the $n \times T$ matrix, oo_.endo_simul, after the stoch_simul command.[2] The n rows of oo_.endo_simul correspond to the n variables in var, *listed in the order in which you have listed them in the var statement* from the first to the last row. To verify the order that Dynare puts the variables in, see how they are ordered in M_.endo_names.

Retrieve output growth from oo_.endo_simul and get the log level of output, y, using the cumsum command in MATLAB. Graph the level of actual output and of natural output. Actual output should appear more volatile than natural output. What is the intuition for this?

(b) Compute the HP filter of y with $\lambda = 1600$. Graph the HP trend, actual output and natural output. Does the HP trend resemble natural output?

(c) Consider the HP filtered output data, i.e., the difference between actual output and its HP trend. This is often used as a measure of the 'output gap'. Graph the output gap computed by HP filter and the actual output gap. Compute their correlation.

(d) Do (a)-(c) with $\sigma_a = 0.00002$. Reset σ_a to 0.02 and replace the time series representation of a_t by the trend stationary representation: $a_t = \rho a_{t-1} + \varepsilon_t^a$. In each case, what happens to the correlation between the actual output gap and the HP-filter based output gap? Provide intuition for what you find. Do the results influence your views about the relative plausibility of difference stationary, versus trend stationary time series representations for shocks? Do they influence your views about the accuracy of the HP filter as a procedure for estimating the output gap?

3. Consider the model in the previous section. Generate 5,000 observations artificial data from the model (i.e., set periods=5000). After the *stoch_simul command*, place the

[2] Here, endo_simul is the matrix, which is a 'field' in the structure, oo_.

simulated data, *oo_.endo_simul*, in the matrix, *sim* (i.e., set *sim=oo_.endo_simul*). Then, save this matrix to a MATLAB file, *data*, using the instruction, *save data sim.* The next step is to launch a Dynare mod file which reads in the data and estimates the four parameters, ϕ_π, σ_a, σ_τ, ρ. That mod file should begin with the usual things: a *var* command with the 7 model variables, a *varexo* command with the two shock innovations, a *parameters* command which lists the names of parameters that are required for the model. After the model command, the parameters to be estimated and their priors should be inserted between a *estimated_params;* and *end;* . I recommend the following:

$$stderr\ eps\_a,\ inv\_gamma\_pdf, 0.02, 10;$$

$$stderr\ eps\_tau,\ inv\_gamma\_pdf, 0.02, 10;$$

$$rho,\ beta\_pdf,\ 0.9, .04;$$

$$lambda,\ beta\_pdf,\ 0.5, .04;$$

where *eps_a*, *eps_tau* correspond to the standard deviation of ε_t^a and ε_t^τ, respectively. Also, *rho* corresponds to the parameter, ρ and *lambda* corresponds to the parameter, λ. My suggestion is to use the beta distribution as the prior for ρ and λ, because that only assigns positive density to the interval, $(0, 1)$. Use the inverted gamma distribution because that constrains the standard deviations to be non-negative (it sort of has the shape of a log-normal). The two numbers after the name of the density function correspond to the mode of the density and the standard deviation, respectively. Note that I suggest starting by setting the modes of the parameters to their true values. The standard deviation of a prior distribution is not something anyone really has any feel for (unless it's the standard deviation of a Normal variable). The best way to evaluate a particular specification of the prior is to look at the graph of the prior distribution that is created by Dynare.

Dynare tries to find the mode of the posterior distribution, but it needs your help, in the form of an initial guess. For that, you must insert instructions between the Dynare command lines, *estimated_params_init;* and *end;* . The instruction in the case of the standard deviation of ε_t^a, is as follows:

$$stderr\ eps\_a,\ 0.02;$$

Note that I'm recommending giving Dynare the true value for the initial guess. Insert a similar command for ε_t^{τ}. In the case of ρ, I suugest the following command:

$$rho,\ 0.9;$$

Note that here too, I'm suggesting giving Dynare the true value of ρ as an initial guess. Insert a similar instruction for λ.

Dynare needs to know which variables in the *var* command will be used in the estimation. This is done with the varobs command:

$$varobs\ pie\ dy,$$

Here, *pie* is inflation (you can use whatever name for this you want, of course) and *dy* is output growth.

You must prepare a MATLAB *.m* file, you could call it *data_place.m,* which has the property that when Dynare executes *data_file*, two vectors, *pie* and *dy*, appears in memory, containing the data that will be used in estimation.

Now you are ready to do estimation! This is accomplished with the command, *estimation*:

$$estimation(datafile = data\_file, , first\_obs = 51, forecast = 36, nobs = 30, mode\_check, mode\_$$

Here, the argument, $datafile = xx$, tells Dynare where to find the data (*datafile=* is a Dynare command, you can't change that). The command, *first_ obs=xx,* tells Dynare to begin working with the 51^{st} observation and *nobs=yy* tells Dynare to use *yy* observations. The parameter, *mode_ check*, tells Dynare to display graphs of the posterior distribution in which one parameter is varied and all others are held fixed at Dynare's approximation of the posterior mode. The graphs generated by *mode_ check* are useful as diagnostic tools to verify that Dynare did indeed find the mode. Finally, *mh_replic* is the length of the MCMC chain; *mh_jscale* is the constant in the jump distribution (i.e., k in the notes); *mh_nblocks* is the number of chains to do; *forecast=36* means forecast all the variables 36 periods into the future; and $mode\_compute = 4$ means use numerical algorithm number 4 (there are about 7 options, but 4 is good) to find the posterior mode.

(a) Now, let's do Bayesian estimation. Set the mean of the priors over the parameters to the corresponding true values. Set the standard deviation of the inverted gamma to 10 and of the beta to 0.04. (It's hard to interpret these standard deviations directly, but you will see graphs of the priors, which are easier to interpret.) Use 30 observations in the estimation. Adjust the value of k, so that you get a reasonable acceptance rate. Dynare generates graphs with the marginal posteriors and priors. How do the priors and posteriors compare. You should notice that there isn't much information in the data about the autocorrelation in τ_t. It is not surprising, given that the data are generated by an NK model, that information about labor supply is hard to pin down. Fluctuations in the markup have the consequence that movements in labor demand are amplified and play a major role in the dynamics.

(b) Redo (a), but set the mean and standard deviation of the prior on λ equal to 0.95 and 0.04, respectively. Does the impression from (a) that there is little evidence in the data change?

(c) Note how the priors on σ_a and ρ have faint 'shoulders' on the right side. Redo (a), with $mh\_replic = 4,000$. Note that the posteriors are now smoother. Actually, $mh\_replic = 4,000$ is a small number of replications to use in practice.

(d) Now set the mean of the priors on the standard deviations to 0.1, far from the truth. Set the prior standard deviation on the inverted gamma distributions to 1. Keep the observations at 30, and see how the posteriors compare with the priors. (Reset $mh\_replic = 4,000$ so that the computations go quickly.) Note that the posteriors move sharply back into the neighborhood of 0.02. Evidently, there is a lot of information in the data about these parameters.

(e) Repeat (a) with 4,000 observations. Display graphs of the priors and posteriors. Note how, with one exception, the posteriors are 'spikes' relative to the priors. The exception, still, is lambda.