Leverage Restrictions in a Business Cycle Model

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We seek to develop a business cycle model with a financial sector, which can be used to study the consequences of policies to restrict the leverage of financial institutions (banks).[1] Because we wish the model to be consistent with basic features of business cycle data, we introduce our banking system into a standard medium sized DSGE model such as Christiano, Eichenbaum and Evans (2005) (hereinafter, CEE) or Smets and Wouters (2007). Banks in our model operate in perfectly competitive markets. Our model implies that social welfare is increased by restricting bank leverage relative to what leverage would be if financial markets were unregulated. With less leverage, banks are in a position to use their net worth to insulate creditors in case there are losses on bank's balance sheets. Our model implies that by reducing risk to creditors, agency problems are mitigated and the efficiency of the banking system is improved. We explore the economics of our result by studying the model's steady state.

1. By "banks" we mean all financial institutions, not just commercial banks.

We also display various dynamic features of the model to assess its empirical plausibility.

There are two types of motivations for restrictions on banking leverage. One motivates leverage restrictions as a device to correct an agency problem in the private economy. Another motivates leverage restrictions as a device to correct a commitment problem in the government.[2] In this paper we focus on the former type of rationale for leverage restrictions.

We posit the existence of an agency problem between banks and their creditors. By bank creditors, we have in mind real-world depositors, holders of debt securities like bonds and commercial paper, and also holders of bank preferred stock.[3] As a result, bank credit in our model is risky. To quantify this risk, we calibrate the model to the premium paid by banks for funds in the interbank market. This premium is on average about 50 basis points at an annual rate.[4] To simplify the

2. For example, Chari and Kehoe (2012) show that a case for leverage restrictions can be built on the assumptions that (i) bankruptices are ex post inefficient and (ii) governments are unable to commit ex ante to not bailout failed banks. See also Gertler, Kiyotaki and Queraltó (2012) for a discussion. In the general discussion of Adrian, Colla and Shin (2013), Robert Hall draws attention to the implications of, for bank leverage decisions, the expectation of government intervention in a crisis episode.

3. Our logic for including bank preferred stock in bank "credit" is as follows. In our model, the liability side of banks' balance sheets has only "bank debt" and "bank net worth." For the vast majority of banks in our model, their asset portfolio performs well enough that debt holders receive a high return, and bank net worth generally earns a positive return. In the case of banks in our model whose portfolio of assets performs poorly, net worth is wiped out and debt holders earn a low return. The reason we think of preferred stock as part of bank debt in the model is: (i) dividend payments on preferred stock are generally not contingent on the overall performance of the bank's assets, unless the performance of the assets is so bad that common stock holders are wiped out; and (ii) like ordinary debt, holders of preferred stock do not enjoy voting rights. Our model abstracts from the differences that do exist between the different components of what we call bank debt. For example, dividends on preferred stock are paid after interest and principal payments on bank's bonds, commercial paper and deposits. In addition, the tax treatment of preferred stock is different from the tax treatment of a bank's bond and commercial paper. The reason we identify the common stock portion of bank liabilities with bank net worth in our model is that holders of common stock are residual claimants. As a result, they are the recipients of increases in bank earnings (magnified by leverage) and they suffer losses when earnings are low (and, these losses are magnified by leverage). Financial firms are very important in the market for preferred stock. For example, Standard and Poor's computes an overall index of the price and yield on preferred stock. In their index for December 30, 2011, 82 percent of the firms belong to the financial sector (see https:// www.sp-indexdata. com/idpfiles/strategy/prc/active/factsheets/fs-sp-us-preferred-stock-index-ltr.pdf).

4. We measure the interest rate on the interbank market by the 3-month London interbank offer rate (Libor). The interest rate premium is the excess of Libor over the 3-month rate on U.S. government Treasury bills.

analysis, we assume there is no agency problem on the asset side of banks' balance sheets. The role of banks in our model is to exert costly effort to identify good investment projects. The source of the agency problem in our model is our assumption that bank effort is not observed. Under these circumstances, it is well known that competitive markets do not necessarily generate the efficient allocations. In our analysis, the fact that banker effort is unobserved has the consequence that restricting the amount of liabilities a bank may issue raises welfare.

As in any model with hidden effort, the resulting agency problem is mitigated if the market provides the agent (i.e., the banker) with the appropriate incentives to exert effort. For this, it is useful if the interest rate that the banker pays to its creditors is not sensitive to the performance of the asset side of its balance sheet. In this case, the banker reaps the full reward of its effort. But, this requires that the banker have sufficient net worth on hand to cover the losses that will occasionally occur even if a high level of effort is expended. The creditors in low net worth banks that experience bad outcomes on their portfolio must necessarily share in bank losses. Understanding this in advance, creditors require that low net worth bankers with well-performing portfolios pay a high interest rate. Under these circumstances, the banker does not enjoy the full fruits of its effort and so its incentive to exert effort is correspondingly reduced.

We analyze the steady state properties of the model and show that a leverage restriction moves equilibrium consumption and employment in the direction of the efficient allocations that would occur if effort were observable. In particular, when banks are restricted in how many liabilities they can issue, then they are more likely to be able to insulate their creditors from losses on the asset side of their balance sheet. In this way, leverage restrictions reduce the interest rate spread faced by banks and promote their incentive to exert effort. We calibrate our model's parameters so that leverage is 20 in the absence of regulation. When a regulation is imposed that limits leverage to 17, steady state welfare jumps to an amount that is equivalent to a permanent 1.19 percent jump in consumption.[5]

After obtaining these results for the steady state of the model, we turn to its dynamic properties. We display the dynamic response

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5. In our analysis, we do not factor in the bureaucratic and other reporting costs of leverage restrictions. If we do so, presumably the steady state welfare benefit of leverage would be smaller. However, because the benefits reported in this paper are so large, we expect our finding that welfare increases to be robust.

of various variables to four shocks. Of these, one is a monetary policy shock, two are shocks to bank net worth and a fourth is a shock to the cross-sectional dispersion of technology.[6] In each case, a contractionary shock drives down consumption, investment, output, employment, inflation and bank net worth, just as in actual recessions. In addition, all four shocks raise the cross-sectional dispersion of bank equity returns. We use the Center for Research on Security Prices (CRSP) data to show that this implication is consistent with the data. The countercyclical nature of various measures of dispersion has been a subject of great interest since Bloom (2009) drew attention to the phenomenon. A factor that may be of independent interest is that our paper provides examples of how this increase in dispersion can occur endogenously.[7]

The paper is organized as follows. The next section describes the circumstances of the bankers. We then describe the general macroeconomic environment into which we insert the bank. After that we report our findings for leverage and for the dynamic properties of our model. The last section includes concluding remarks.

1. BANKS, MUTUAL FUNDS AND ENTREPRENEURS

We begin the discussion in period t, after goods production for that period has occurred. There is a mass of identical bankers with net worth N_t. The bankers enter into competitive and anonymous markets, acquire deposits from mutual funds and lend their net worth and deposits to entrepreneurs. Mutual funds take deposits from households and make loans to a diversified set of banks. The assumption that mutual funds stand between households and banks is made for convenience. Our bankers are risky and if households placed deposits directly with banks they would choose to diversify across banks. The idea that households diversify across a large set of banks seemed awkward to us. Instead, we posit that households hold deposits with mutual funds, and then mutual funds diversify across banks. Another advantage of our assumption that mutual

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6. For the latter we consider a risk shock, which is similar to the one considered in Christiano, Motto and Rostagno (2014).
7. For examples in which exogenous fluctuations in uncertainty can account for a substantial fraction of business cycle fluctuations, see Bloom (2009) and Christiano, Motto and Rostagno (2014).

funds stand between households and banks is that this allows us to define a risk-free rate of interest. However, nothing of substance hinges on the presence of the mutual funds.

Each entrepreneur has access to a constant returns-to-scale investment technology. The technology requires, as input, an investment at the end of goods production in period t and produces output during production in $t + 1$. Entrepreneurs are competitive, earn no rent and there is no agency problem between entrepreneurs and banks. The bank from which an entrepreneur receives its loan receives the full rate of return earned by entrepreneurs on their projects.

There are "good" and "bad" entrepreneurs. We denote the gross rate of return on their period t investment by R_{t+1}^g and R_{t+1}^b respectively, where $R_{t+1}^g > R_{t+1}^b$ in all period $t + 1$ states of nature. These represent exogenous stochastic processes from the point of view of entrepreneurs. We discuss the factors that determine these rates of return in the next section. There, we situate entrepreneurs and bankers in the broader macro economy.

A key function of banks is to identify good entrepreneurs. To do this, bankers exert a costly effort. In our baseline model this effort is not observable to the mutual funds that supply the banks with funds, and this creates an agency problem on the liability side of a bank's balance sheet. As a convenient benchmark, we also consider the version of the model in which banker effort is observable to the mutual fund that supplies the bank with deposits d_t.

At the end of production in period t, each banker takes deposits d_t and makes loans in the amount $N_t + d_t$ to entrepreneurs. We capture the idea that banks are risky with the assumption that a bank can only invest in one entrepreneur.[8] The quantities N_t and d_t are expressed in per capita terms.

We denote the effort exerted by a banker to find a good entrepreneur by e_t. The banker identifies a good entrepreneur with probability $p(e_t)$ and a bad entrepreneur with the complementary

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8. We can describe the relationship between a bank and an entrepreneur in search theoretic terms. Thus, the bank exerts an effort e_t to find an entrepreneur. Upon exerting this effort a bank meets exactly one entrepreneur in a period. We imagine that the outside option for both the banker and the entrepreneur at this point is zero. We suppose that upon meeting, the bank has the option to make a take-it-or-leave-it offer to the entrepreneur. Under these circumstances, the bank will make an offer that puts the entrepreneur on its outside option of zero. In this way, the banker captures all the rent in their relationship.

probability. For computational simplicity, we adopt the following simple representation of the probability function:

$$p(e) = \min\{1, \overline{a} + \overline{b}e\}, \ \ \overline{a}, \overline{b} \geq 0$$

Because we work with equilibria in which $p(e_t) > 1/2$, our model implies that when bankers exert greater effort, the mean return on their asset increases and its variance decreases.

Mutual funds are competitive and perfectly diversified across good and bad banks. As a result of free entry, they enjoy zero profits:

$$p(e_t) R_{g,t+1}^d + (1 - p(e_t)) R_{b,t+1}^d = R_t \quad\quad\quad (1)$$

in each period $t + 1$ state of nature. Here, $R_{g,t+1}^d$ and $R_{b,t+1}^d$ denote the gross return received from good and bad banks, respectively. In (1), $p(e_t)$ is the fraction of banks with good returns, and $1 - p(e_t)$ is the fraction of banks with bad returns.[9] The following two subsections discuss the deposit contracts between banks and mutual funds that emerge in equilibrium. The first discussion reviews the case when mutual funds observe e_t. The case that we consider empirically relevant is the one in which the e_t selected by a bank is not observed by the mutual fund that provides the bank with deposits. The latter case is considered in the subsequent section. After that we describe the aggregate law of motion of banker net worth. Finally, we describe the changes to the environment when there are binding leverage restrictions.

1.1 Deposit Contracts When Banker Effort is Observable

A loan contract between a banker and a mutual fund is characterized by four objects,

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9. We obtain (1) as follows. The period t measure of profits for mutual funds is

$$E_t \lambda_{t+1} [p(e_t) R_{g,t+1}^d + (1 - p(e_t)) R_{b,t+1}^d - R_t],$$

where the product of λ_{t+1} and the associated conditional probability is proportional to the state contingent price of cash. In addition, we assume the only source of funds for mutual funds in period $t + 1$ is the revenues from banks, so that mutual funds have the following state-by-state non-negativity constraint:

$$p(e_t) R_{g,t+1}^d + (1 - p(e_t)) R_{b,t+1}^d - R_t \geq 0.$$

Equation (1) is implied by the zero profit condition and the above non-negativity constraint.

$$(d_t, e_t, R^d_{g,t+1}, R^d_{b,t+1}). \tag{2}$$

In this section, all four elements of the contract are assumed to be directly verifiable by the mutual fund. Throughout this paper, we assume that sufficient sanctions exist so that verifiable deviations from a contract never occur.

The representative mutual fund takes R_t as given. We assume the banker's only source of funds for repaying the mutual fund is the earnings on its investment. Regardless of the return on its asset, the banker must earn enough to pay its obligation to the mutual fund:

$$R^g_{t+1}(N_t + d_t) - R^d_{g,t+1}d_t \geq 0, \quad R^b_{t+1}(N_t + d_t) - R^d_{b,t+1}d_t \geq 0.$$

Mutual funds are obviously only interested in contracts that are feasible, so the above inequalities represent restrictions on the set of contracts that mutual funds are willing to consider. In practice, only the second inequality is ever binding.

In equilibrium, each bank has access to a menu of contracts, defined by the objects in (2) which satisfy (1) and

$$R^b_{t+1}(N_t + d_t) - R^d_{b,t+1}d_t \geq 0. \tag{3}$$

as well as non-negativity of e_t and d_t. The problem of the banker is to select a contract from this menu.

A banker's ex-ante reward from a loan contract is:

$$E_t\lambda_{t+1}\{p(e_t)\,[R^g_{t+1}(N_t + d_t) - R^d_{g,t+1}d_t]$$
$$+ (1 - p(e_t))[R^b_{t+1}(N_t + d_t) - R^d_{b,t+1}d_t]\} - \frac{1}{2}\,e_t^2, \tag{4}$$

where $e_t^2/2$ is the banker's utility cost of expending effort and λ_{t+1} denotes the marginal value of profits to the household. As part of the terms of the banker's arrangement with its own household, the banker is required to seek a contract that maximizes (4).[10] Formally, the banker maximizes (4) by choice of $e_t, d_t, R^d_{g,t+1}$, and $R^d_{b,t+1}$ subject

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10. Throughout the analysis we assume the banker's household observes all the variables in (4) and that the household has the means (say, because the household could threaten to withhold the perfect consumption insurance that it provides) to compel the banker to do what the household requires of it.

to (1) and (3). In appendix A of the working paper[11], we show that (3) is non-binding and that the following are the optimization conditions:

$$e \quad : \quad e_t = E_t \lambda_{t+1} p_t'(e_{t+1}) (R_{t+1}^g - R_{t+1}^b)(N_t + d_t) \tag{5}$$

$$d \quad : \quad E_t \lambda_{t+1}[p_t(e_t) R_{t+1}^g + (1 - p_t(e_t)) R_{t+1}^b - R_t] = 0 \tag{6}$$

$$\mu \quad : \quad R_t = p_t(e_t) R_{g,t+1}^d + (1 - p_t(e_t)) R_{b,t+1}^d. \tag{7}$$

Here, the character before the colon indicates the variable being differentiated in the Lagrangian version of the bank's optimization problem. The character μ denotes the multiplier on (1). Note from (5) how the size of the base $N_t + d_t$ on which banks make profits affects effort e_t. Also, note from (5) that in setting effort e_t, the banker looks only at the sum $N_t + d_t$, and not at how this sum breaks down into the component reflecting the banker's own resources N_t and the component reflecting the resources d_t supplied by the mutual fund. By committing to care for d_t as if these were the banker's own funds, the banker is able to obtain better contract terms from the mutual fund. The banker is able to commit to the level of effort in (5) because e_t is observable to the mutual fund.

The values of the state contingent return on the deposits of banks with good and bad investments $R_{g,t+1}^d, R_{b,t+1}^d$ are not uniquely pinned down. These returns are restricted only by (7) and (3). For example, the following scenario is compatible with the equations $R_{g,t+1}^d = R_{t+1}^g$, $R_{b,t+1}^d = R_{t+1}^b$. It may also be possible for the equations to be satisfied by a non-state contingent pattern of returns, $R_{g,t+1}^d = R_{b,t+1}^d = R_t$. However, (3) indicates that the latter case requires N_t to be sufficiently large.

1.2 Deposit Contracts When Banker Effort is Not Observable

We now suppose that the banker's effort, e_t is not observed by the mutual fund. Thus, whatever $d_t, R_{g,t+1}^d, R_{b,t+1}^d$ and e_t are specified in the contract, a banker always chooses e_t ex post to maximize (4). The first order condition necessary for optimality is:

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11. Working Papers of the Central Bank of Chile: http://www.bcentral.cl/Estudios/documentos-trabajo/fichas/726.htm

$$e \quad : \quad e_t = E_t \lambda_{t+1} p_t'(e_t)[(R_{t+1}^g - R_{t+1}^b)(N_t + d_t) - (R_{g,t+1}^d - R_{b,t+1}^d)d_t]. \quad (8)$$

Note that $R_{g,t+1}^d > R_{b,t+1}^d$ reduces the banker's incentive to exert effort. This is because, in this case, the banker receives a smaller portion of the marginal increase in expected profits caused by a marginal increase in effort. The representative mutual fund understands that e_t will always be selected according to (8). Since the mutual fund is only interested in contracts that will actually be implemented, it will only offer contracts that satisfy not just (3), but also (8). Thus, we assume that the menu of contracts that exists in equilibrium is the set of $(d_t, e_t, R_{g,t+1}^d, R_{b,t+1}^d)$'s that satisfy (1), (3) and (8). The banker's problem now is to maximize (4) subject to these three conditions. In the appendix, we show that the conditions for optimization are:

$$e \quad : \quad E_t(\lambda_{t+1} + \nu_{t+1}) p_t'(e_t)(R_{g,t+1}^d - R_{b,t+1}^d) d_t + \eta_t = 0 \quad (9)$$

$$d \quad : \quad 0 = E_t(\lambda_{t+1} + \nu_{t+1})[p_t(e_t)(R_{t+1}^g - R_{g,t+1}^d) + (1 - p_t(e_t))(R_{t+1}^b - R_{b,t+1}^d)]$$

$$R_g^d : \quad \nu_{t+1} p_t(e_t) + \eta_t \lambda_{t+1} p_t'(e_t) = 0$$

$$\mu \quad : \quad R_t = p_t(e_t) R_{g,t+1}^d + (1 - p_t(e_t)) R_{b,t+1}^d$$

$$\eta \quad : \quad e_t = E_t \lambda_{t+1} p_t'(e_t)[(R_{t+1}^g - R_{t+1}^b)(N_t + d_t) - (R_{g,t+1}^d - R_{b,t+1}^d) d_t]$$

$$\nu \quad : \quad \nu_{t+1}[R_{t+1}^b(N_t + d_t) - R_{b,t+1}^d d_t] = 0, \ \nu_{t+1} \geq 0,$$

$$[R_{t+1}^b(N_t + d_t) - R_{b,t+1}^d d_t] \geq 0.$$

Here, η_t is the multiplier on (8), ν_{t+1} is the multiplier on (3). The date on a multiplier indicates the information on which it is contingent. Thus, η_t, ν_t and μ_t are each contingent on the period t realization of aggregate shocks. For computational simplicity, we only consider parameter values such that the cash constraint (3) is always binding. The first three equations in (9) correspond to first order conditions associated with the Lagrangian representation of the banker problem, with the names corresponding to the variable being differentiated.

The magnitude of the multiplier, $\nu_{t+1} \geq 0$, is a measure of the inefficiency of the banking system. If ν_{t+1} is zero, then $\eta_t = 0$ is zero by the R_g^d condition in (9). Then, combining the e equation with the

η equation, we see that e_t is set efficiently, in the sense that it is set according to (5). When $v_{t+1} > 0$ then $\eta_t < 0$ and e_t is below the level indicated by (5).[12]

A notable feature of the model concerns its implication for the cross-sectional variance on the rate of return on bank equity. In period $t + 1$ the realized rate of return on bank equity for the $p(e_t)$ successful banks and for the $1 - p(e_t)$ unsuccessful banks is, respectively,

$$\frac{R_{t+1}^g \left(N_t + d_t \right) - R_{g,t+1}^d d_t}{N_t}, \frac{R_{t+1}^b \left(N_t + d_t \right) - R_{b,t+1}^d d_t}{N_t}.$$

Given our assumption that the cash constraint is binding for unsuccessful banks, the second of the above two returns is zero. So, the period t cross-sectional standard deviation s_{t+1}^b and mean E_{t+1}^b of bank equity returns are:[13]

$$s_{t+1}^b = \left[p\left(e_t \right)\left(1 - p\left(e_t \right) \right) \right]^{1/2} \frac{R_{t+1}^g \left(N_t + d_t \right) - R_{g,t+1}^d d_t}{N_t}, \tag{10}$$

$$E_{t+1}^b = p\left(e_t \right) \frac{R_{t+1}^g \left(N_t + d_t \right) - R_{g,t+1}^d d_t}{N_t}.$$

When e_t increases, banks become safer in the sense that their Sharpe ratio E_{t+1}^b / s_{t+1}^b increases.

1.3 Law of Motion of Aggregate Bank Net Worth

In the next section, we assume that each banker is a member of one of a large number of identical households. Each household has sufficiently enough bankers that the law of large numbers applies. We assume that the bankers in period t all have the same level of net worth, N_t. We assume in $t + 1$ they pool their net worth after their period $t + 1$ returns are realized. In this way, we avoid the potentially distracting problem of having to model the evolution of the

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12. In appendix A we show that v_{t+1} is positive in any period $t + 1$ state of nature if, and only if, it is positive in all period $t + 1$ states of nature.

13. Recall that if a random variable has a binomial distribution and takes on the value x^h with probability p and x^l with probability $1 - p$, then the variance of that random variable is $p(1 - p)(x^h - x^l)^2$.

distribution of banker net worth. After bankers have pooled their net worth in period $t + 1$ an exogenous fraction $1 - \gamma_{t+1}$ of this net worth is transferred to their household. At this point, the representative household makes an exogenous lump sum transfer N_{t+1} to the net worth of its banker. After pooling and transfers, the net worth of a banker in the representative household in period $t + 1$ is given by:

$$N_{t+1} = \gamma_{t+1} \left\{ \begin{array}{l} p(e_t)\left[R_{t+1}^g\left(N_t + d_t \right) - R_{g,t+1}^d d_t \right] \\ +(1 - p(e_t))\left[R_{t+1}^b\left(N_t + d_t \right) - R_{b,t+1}^d d_t \right] \end{array} \right\} + T_{t+1}. \qquad (11)$$

We assume that γ_{t+1} and T_{t+1} are exogenous shocks, realized in $t + 1$. A rise in T_{t+1} is equivalent to an influx of new equity into the banks. Similarly, a rise in γ_{t+1} also represents a rise in equity. Thus, we assume that the inflow or outflow of equity into the banks is exogenous and is not subject to the control of the banker. The only control bankers have over their net worth operates through their control over deposits and the resulting impact on their earnings.

In the unobserved effort model, where we assume the cash constraint is always binding in the bad state, we have:

$$N_{t+1} = \gamma_{t+1} p(e_t)\left[R_{t+1}^g(N_t + d_t) - R_{g,t+1}^d d_t \right] + T_{t+1}. \qquad (12)$$

The object in square brackets is the realized profits of good banks. It is possible for those to make losses on their deposits (i.e., $R_{t+1}^g < R_{g,t+1}^d$), however we assume that those profits are never so negative that earnings on net worth cannot cover them.

When there is no aggregate uncertainty, the d and μ equations (9) imply that the expected earnings of a bank on deposits is zero. Then,

$$p_t(e_t) R_{t+1}^g + (1 - p_t(e_t)) R_{t+1}^b = R_t. \qquad (13)$$

Equation (13) and the μ equation in (9) together imply that the law of motion has the following form:

$$N_{t+1} = \gamma_{t+1} R_t N_t + T_{t+1}. \qquad (14)$$

When there is aggregate uncertainty, equation (13) holds only in expectation. It does not hold in terms of realized values.

1.4 Restrictions on Bank Leverage

We now impose an additional constraint on banks, which they must satisfy:

$$\frac{N_t + d_t}{N_t} \leq L_t, \tag{15}$$

where L_t denotes the period t restriction on leverage. The banker problem now is to maximize (4) subject to (1), (3), (8) and the additional constraint $N_t L_t - (N_t + d_t) \geq 0$. Let $\Lambda_t \geq 0$, denote the multiplier on that constraint. It is easy to verify that the equilibrium conditions now are (9) with the zero in the d equation replaced by Λ_t, plus the following complementary slackness condition:

$$\Lambda_t \left[N_t L_t - (N_t + d_t) \right] = 0, \quad \Lambda_t \geq 0, \quad N_t L_t - (N_t + d_t) \geq 0.$$

Thus, when the leverage constraint is binding, we use the d equation to define Λ_t and add the equation

$$N_t L_t = (N_t + d_t).$$

Interestingly, since the d equation does not hold any longer with $\Lambda_t = 0$, the expected profits of banks in steady state are positive. As a result, (14) does not hold in steady state. Of course, (11) and (12) both hold. Using the μ equation to simplify (11):

$$N_{t+1} = \gamma_{t+1} \{ [p_t(e_t) R_{t+1}^g + (1 - p_t(e_t)) R_{t+1}^b] (N_t + d_t) - R_t d_t] + T_{t+1}. \tag{16}$$

The modified d equation in the version of the model without aggregate uncertainty is:

$$\Lambda_t = (\lambda_{t+1} + \nu_{t+1}) [p_t(e_t)(R_{t+1}^g - R_{g,t+1}^d) + (1 - p_t(e_t)) (R_{t+1}^b - R_{b,t+1}^d)]. \tag{17}$$

Substituting this into (16):

$$N_{t+1} = \gamma_{t+1} \left\{ \left[\frac{\Lambda_t}{\lambda_{t+1} + \nu_{t+1}} + R_t \right] \left(N_t + d_t \right) - R_t d_t \right\} + T_{t+1},$$

or

$$N_{t+1} = \gamma_{t+1} \left\{ R_t N_t + \left[\frac{\Lambda_t}{\lambda_{t+1} + \nu_{t+1}} \right] (N_t + d_t) \right\} + T_{t+1}.$$

From here we see that banks make profits on deposits when the leverage constraint is binding, so that $\Lambda_t > 0$.

2. THE GENERAL MACROECONOMIC ENVIRONMENT

In this section, we place the financial markets of the previous section into an otherwise standard macro model, along the lines of Christiano, Eichenbaum and Evans (2005) or Smets and Wouters (2007). The financial market has two points of contact with the broader macroeconomic environment. First, the rates of return on entrepreneurial projects are a function of the rate of return on capital. Second, there is a market clearing condition in which the total purchases of raw capital by entrepreneurs $N_t + d_t$ is equal to the total supply of raw capital by capital producers. In the following two subsections, we first describe goods production and the problem of households. The second subsection describes the production of capital and its links to the entrepreneur. Later subsections describe monetary policy and other aspects of the macro model.

2.1 Goods Production

Goods are produced according to a Dixit-Stiglitz structure. A representative, competitive final goods producer combines intermediate goods $Y_{j,t}, j \in [0,1]$, to produce a homogeneous good Y_t using the following technology:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{1}{\lambda_f}} dj \right]^{\lambda_f}, 1 \leq \lambda_f < \infty. \tag{18}$$

The intermediate good is produced by a monopolist using the following technology:

$$Y_{j,t} = \begin{cases} \bar{K}_{j,t}^{\alpha} \left(z_t l_{j,t} \right)^{1-\alpha} - \Phi z_t^* & \text{if } \bar{K}_{j,t}^{\alpha} \left(z_t l_{j,t} \right)^{1-\alpha} > \Phi z_t^* \\ 0 & \text{otherwise} \end{cases}, 0 < \alpha < 1. \quad (19)$$

Here, z_t follows a determinist time trend. Also, $\bar{K}_{j,t}$ denotes the services of capital and $l_{j,t}$ denotes the quantity of homogeneous labor, respectively, hired by the jth intermediate good producer. The fixed cost in the production function (19), is proportional to z_t^* which is discussed below. The variable z_t^* has the property that Y_t/z_t^* converges to a constant in non-stochastic steady state. The monopoly supplier of $Y_{j,t}$ sets its price $P_{j,t}$ subject to Calvo-style frictions. Thus, in each period t a randomly selected fraction of intermediate good firms $1 - \xi_p$ can re-optimize their price. The complementary fraction sets its price as follows:

$$P_{j,t} = \pi P_{j,t-1}.$$

Let π_t denote the gross rate of inflation P_t/P_{t-1}, where P_t is the price of Y_t. Then, π denotes the steady state value of inflation.

There exists a technology that can be used to convert homogeneous goods into consumption goods C_t one-for-one. Another technology converts a unit of homogenous goods into investment goods Υ^t, where $\Upsilon > 1$. This parameter allows the model to capture the observed trend fall in the relative price of investment goods. Because we assume these technologies are operated by competitive firms, the equilibrium prices of consumption and investment goods are P_t and

$$P_{I,t} = \frac{P_t}{\Upsilon^t},$$

respectively. The trend rise in technology for producing investment goods is the second source of growth in the model, and

$$z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha} \right) t}.$$

Our treatment of the labor market follows Erceg, Henderson and Levin (2000), and parallels the Dixit-Stiglitz structure of goods production. A representative, competitive labor contractor aggregates

the differentiated labor services $h_{i,t}$ and $i \in [0,1]$ into homogeneous labor l_t using the following production function:

$$l_t = \left[\int_0^1 \left(h_{i,t} \right)^{\frac{1}{\lambda_w}} di \right]^{\lambda_w}, 1 \leq \lambda_w < \infty. \tag{20}$$

The labor contractor sells labor services l_t to intermediate good producers for a given nominal wage rate W_t. The labor contractor also takes as given the wages of the individual labor types $W_{i,t}$.

A representative, identical household supplies each of the differentiated labor types $h_{i,t}$ and $i \in [0,1]$, used by the labor contractors. By assuming that all varieties of labor are contained within the same household (this is the "large family" assumption introduced by Andolfatto, 1996 and Merz, 1995) we avoid confronting difficult distributional issues. For each labor type $i \in [0,1]$, there is a monopoly union that represents workers of that type belonging to all households. The ith monopoly union sets the wage rate $W_{i,t}$ for its members, subject to Calvo-style frictions. In particular, a randomly selected subset of $1 - \xi_w$ monopoly unions set their wage to optimize household utility (see below), while the complementary subset sets the wage according to:

$$W_{i,t} = \mu_{z^*} \pi W_{i,t-1}$$

Here, μ_{z^*} denotes the growth rate of z_t^*. The wage rate determines the quantity of labor demanded by the competitive labor aggregators. Households passively supply the quantity of labor demanded.

2.2 Households

The representative household is composed of a unit measure of agents. Of these, a fraction ρ are workers and the complementary fraction are bankers. Per capita household consumption is C_t, which is distributed equally to all household members. Average period utility across all workers is given by:

$$\log(C_t - b_u C_{t-1}) - \tilde{\psi}_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1 + \sigma_L} di, \tilde{\psi}_L, \sigma_L \geq 0.$$

The object $b_u \geq 0$ denotes the parameter controlling the degree of habit persistence. The period utility function of a banker is:

$$\log(C_t - b_u C_{t-1}) - \tilde{\rho} e_t^2, \tilde{\rho} \equiv \frac{1}{2(1-\rho)}. \tag{21}$$

The representative household's utility function is the equally-weighted average across the utility of all the workers and bankers:

$$\log(C_t - b_u C_{t-1}) - \psi_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1+\sigma_L} di - \frac{1}{2} e_t^2, \psi_L \equiv \rho \tilde{\psi}_L.$$

The representative household's discount value of a stream of consumption, employment and effort is valued as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t - b_u C_{t-1}) - \psi_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1+\sigma_L} di - \frac{1}{2} e_t^2 \right\}, \psi_L, b_u, \sigma_L > 0. \tag{22}$$

Bankers behave as described in section 2. They are assumed to do so in exchange for the perfect consumption insurance received from households. Although the mutual funds from which bankers obtain deposits do not observe banker effort e_t, we assume that a banker's own household observes everything that it does. By instructing the bankers to maximize expected net worth (taking into account their own costs of exerting effort), the household maximizes total end-of-period banker net worth.[14]

The representative household takes e_t and labor earnings as given. It chooses C_t and the quantity of a nominal bond B_{t+1}, to maximize (22) subject to the budget constraint:

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14. A brief observation about units of measure: We measure the financial objects that the banker works with, N_t and d_t in per capita terms. Bankers are a fraction $1 - \rho$ of the population, so that in per banker terms, bankers work with $N_t/(1 - \rho)$ and $d_t/(1 - \rho)$. We assume the banker values profits net of the utility cost of its effort as follows:

$$E_t \lambda_{t+1} \left\{ p(e_t) \left[R_{t+1}^g \left(\frac{N_t + d_t}{1-\rho} \right) - R_{g,t+1}^d \frac{d_t}{1-\rho} \right] + (1 - p(e_t)) \left[R_{t+1}^b \left(\frac{N_t + d_t}{1-\rho} \right) - R_{b,t+1}^d \frac{d_t}{1-\rho} \right] \right\} - \tilde{\rho} e_t^2.$$

Multiplying this expression by $1 - \rho$ and using (21), we obtain (4).

$$P_t C_t + B_{t+1} \leq \int_0^1 W_{i,t} h_{i,t} di + R_t B_t + \Pi_t.$$

Here, Π_t denotes lump sum transfers of profits from intermediate good firms and bankers and taxes. In addition, the household has access to a nominally non-state contingent one-period bond with gross payoff R_t in period $t + 1$. Loan market clearing requires that, in equilibrium:

$$B_t = d_t. \tag{23}$$

2.3 Monetary Policy

We express the monetary authority's policy rule directly in linearized form:

$$R_t - R = \rho_p \left(R_{t-1} - R \right) + \left(1 - \rho_p \right) \begin{bmatrix} \alpha_\pi \left(\pi_{t+1} - \pi \right) \\ +\alpha_{\Delta y} \frac{1}{4} \left(g_{y,t} - \mu_{z^*} \right) \end{bmatrix} + \frac{1}{400} \varepsilon_t^p, \tag{24}$$

where ε_t^p is a shock to monetary policy and ρ_p is a smoothing parameter in the policy rule. Here, $R_t - R$ is the deviation of the period t net quarterly interest rate R_t from its steady state. Similarly, $\pi_{t+1} - \pi$ is the deviation of anticipated quarterly inflation from the central bank's inflation target. The expression $g_{y,t} - \mu_{z^*}$ is quarterly GDP growth, in deviation from its steady state. Finally, ε_t^p is an *iid* shock to monetary policy with standard deviation σ_p. Note that the shock is in units of annual percentage points.

2.4 Capital Producers, Entrepreneurial Returns and Market Clearing Conditions

In this section we explain how entrepreneurial returns are linked to the underlying return on physical capital. In addition, we discuss the agents that produce capital, the capital producers. Finally, we present the final goods market clearing condition and the market clearing for capital.

The sole source of funds available to an entrepreneur is the funds $N_t + d_t$ received from its bank after production in period t.

An entrepreneur uses these funds to acquire raw capital, $\tilde{K}_{t+1}$ and convert it into effective capital units,

$$P_{k',t} \tilde{K}_{t+1} = N_t + d_t$$

where $P_{k',t}$ is the nominal price of a unit of new, raw capital. This is the market clearing condition for capital. Good and bad entrepreneurs convert one unit of raw capital into e^{g_t}, e^{b_t}, units of effective capital, respectively, where $g_t > b_t$. Once this conversion is accomplished, entrepreneurs rent their homogeneous effective capital into the $t + 1$ capital market. Thus, in period $t + 1$ the quantity of effective capital is K_{t+1} where

$$\bar{K}_{t+1} = [p(e_t) e^{g_t} + (1 - p(e_t)) e^{b_t}] \tilde{K}_{t+1}. \tag{25}$$

Here, e_t is the level of effort expended by the representative banker in period t. Note that if e_t is low in some period, then the effective stock of capital is low in period $t + 1$. This reduction has a persistent effect, because—as we shall see below—effective capital is the input into the production of new raw capital in later periods. This effect of banker effort into the quantity of effective capital reflects their role in allocating capital between good and bad entrepreneurs. The object in square brackets in (25) resembles the "capital destruction shock" adopted in the literature, though here it is an endogenous variable. We refer to it as a measure of the allocative efficiency of the banking system.

Entrepreneurs rent the services of effective capital in a competitive, period $t + 1$ capital market. The equilibrium nominal rental rate in this market is denoted by $P_{t+1} r^k_{t+1}$.[15] Entrepreneurs' effective capital $\bar{K}_{t+1}$ depreciates at the rate δ while it is being used by firms to produce output. The nominal price at which entrepreneurs sell used effective capital to capital producers is denoted $P_{k,t+1}$. The rates of return enjoyed by good and bad entrepreneurs are given by:

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15. Here, the real rental rate on capital has been scaled. That actual real rental rate of capital is $r^k_{t+1} \Upsilon^{-t-1}$. The latter is a stationary object, according to the model. In the model, the rental rate of capital falls in steady state because the capital stock grows at a rate faster than z_t due to the trend growth in the productivity of making investment goods.

$$R_{t+1}^g = e^{g_t} R_{t+1}^k, R_{t+1}^b = e^{b_t} R_{t+1}^k, \tag{26}$$

where

$$R_{t+1}^k \equiv \frac{r_{t+1}^k \Upsilon^{-t-1} P_{t+1} + \left(1 - \delta\right) P_{k,t+1}}{P_{k',t}}.$$

Here, R_{t+1}^k is a benchmark return on capital. The actual return enjoyed by entrepreneurs scales the benchmark according to whether the entrepreneur is good or bad.

We assume there are a large number of identical capital producers. The representative capital producer purchases the time t stock of effective capital and time t investment goods I_t and produces new, raw capital using the following production function:

$$\tilde{K}_{t+1} = (1 - \delta) \bar{K}_t + (1 - S(I_t/I_{t-1})) I_t, \tag{27}$$

where S is an increasing and convex function defined below. The number of capital producers is large enough that they behave competitively. However, there is no entry or exit by entrepreneurs in order to avoid complications that would otherwise arise due to the presence of lagged investment in the production function for new capital. The representative capital producer takes the price of "old" effective capital $P_{k,t}$ as given, as well as the price of new, raw capital $P_{k't}$. If we denote the amount of effective capital that the capital producer purchases in period t by x_t, and the amount of raw capital that it sells in period t by y_t, then its objective is to maximize:

$$\sum_{j=0}^{\infty} \lambda_{t+j} \left\{ P_{k',t+j} y_{t+j} - P_{k,t+j} x_{t+j} - P_{I,t+j} I_{t+j} \right\},$$

where λ_t denotes the multiplier on the household budget constraint and $P_{I,t}$ denotes the price of investment goods. The multiplier and the prices are denominated in money terms. Substituting out for y_t using the production function, we obtain:

$$\max_{\{x_{t+j}, I_{t+j}\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \lambda_{t+j} \left\{ \begin{array}{l} P_{k',t+j} \left[x_{t+j} + \left(1 - S\left(I_{t+j} / I_{t+j-1}\right)\right) I_{t+j} \right] \\ -P_{k,t+j} x_{t+j} - P_{I,t+j} I_{t+j} \end{array} \right\}$$

From this expression, we see that the capital producer will set $x_t = \infty$ if $P_{k',t} < P_{k,t}$ or set $x_t = 0$ if $P_{k',t} < P_{k,t}$. Since neither of these conditions can hold in equilibrium, we conclude that

$P_{k',t} = P_{k,t}$ for all t.

Thus, the problem is simply to choose I_{t+j}, to maximize:

$$\lambda_t \{ P_{k',t}[(1 - S(I_t/I_{t-1})) I_t] - P_{I,t} I_t \}$$

$$+ E_t \lambda_{t+1} \{ P_{k',t+1}(1 - S(I_{t+1}/I_t)) I_{t+1} - P_{I,t+1} I_{t+1} \} + \ ..$$

The first order necessary condition for a maximum is:

$$\lambda_t \left[P_{k',t} \left(1 - S\left(I_t / I_{t-1} \right) - S'\left(I_t / I_{t-1} \right) \frac{I_t}{I_{t-1}} \right) - P_{I,t} \right]$$

$$+ E_t \lambda_{t+1} P_{k',t+1} S'\left(I_{t+1} / I_t \right) \left(\frac{I_{t+1}}{I_t} \right)^2 = 0. \tag{28}$$

Market clearing in the market for old capital requires:

$$x_t = (1 - \delta) \, \bar{K}_t.$$

Combining (27) with (25), we have the equilibrium law of motion for capital:

$$\bar{K}_{t+1} = [\, p_t(e_t) \, e^{g_t} + (1 \, p_t(e_t)) \, e^{b_t}] \, [(1 - \delta) \, \bar{K}_t + (1 - S(I_t/I_{t-1})) \, I_t].$$

Finally, we have the market clearing condition for final goods Y_t, which is:

$$Y_t = G_t + C_t + \frac{I_t}{\Upsilon^t}.$$

2.5 Shocks, Adjustment Costs, Resource Constraint

The adjustment cost function on investment is specified as follows:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \left(\exp\left[\frac{1}{2}\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - \mu_{z^*}\Upsilon\right)\right] + \exp\left[-\frac{1}{2}\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - \mu_{z^*}\Upsilon\right)\right] - 2\right),$$

where the parameter S'' controls the curvature of the adjustment cost function. Also, we specify that T_t and G_t evolve as follows:

$$T_t = z_t^*\tilde{T}_t, \quad G_t = z_t^*\tilde{g}_t,$$

where $\tilde{g}$ is a parameter and the additive equity shock $\tilde{T}_t$ obeys the following law of motion:

$$\log\left(\tilde{T}_t/\tilde{T}\right) = \rho_T \log\left(\tilde{T}_{t-1}/\tilde{T}\right) - \varepsilon_t^T.$$

The multiplicative equity shock, γ_t, obeys the following law of motion:

$$\log\left(\gamma_t/\gamma\right) = \rho_\gamma \log\left(\gamma_{t-1}/\gamma\right) - \varepsilon_t^\gamma.$$

Our third financial shock is a risk shock, Δ_t, which is similar to the one considered in Christiano, Motto and Rostagno (2014). In particular, let

$$b_t = b - \Delta_t$$

$$g_t = g.$$

Thus, Δ_t is a shock to the return to bad banks.
We assume

$$\Delta_t = \rho_\Delta \Delta_{t-1} + \varepsilon_t^\Delta.$$

The innovations to our three financial shocks are iid and

$$E(\varepsilon_t^T)^2 = (\sigma_T)^2, \quad E(\varepsilon_t^\gamma)^2 = (\sigma_\gamma)^2, \quad E(\varepsilon_t^\Delta)^2 = (\sigma_\Delta)^2.$$

3. RESULTS

We first consider the steady state implications of our model for leverage. We then turn to the dynamic implications.

3.1 Model Parameterization

Our baseline model is the one in which banker effort is not observable and there are no leverage restrictions on banks. There are four shock processes, and these are characterized by 7 parameters

$$\sigma_p = 0.25, \quad \sigma_T = \sigma_\gamma = 0.01, \quad \sigma_\Delta = 0.05$$

$$\rho_T = \rho_\gamma = \rho_\Delta = 0.95.$$

The monetary policy shock is in annualized percentage points. Thus, its standard deviation is 25 basis points. Two of the other three shocks are in percent terms. Thus, the innovation to the equity shocks is 1 percent each, and the innovation to risk is 0.1 percent. The autocorrelations are 0.95 in each.

Apart from the parameters of the shock processes, that model has the 25 parameters displayed in table 1. Among these parameters, values for these eight:

$$b, g, \bar{a}, \tilde{T}, \tilde{g}, \Phi, \mu_{z*}, \Upsilon,$$

where chosen to hit the eight calibration targets listed in table 2.

The first calibration target in table 2 is based on the evidence in figure 1. That figure reproduces data constructed in Ferreira (2012). Each quarterly observation in the figure is the cross-sectional standard deviation of the quarterly rate of return on equity for financial firms in the CRSP data base. The sample mean of those observations is 0.2, after rounding. The analog in our model of the volatility measure in figure 1 is s^b in (10). We calibrate the model so that in steady state $s^b = 0.20$. The cyclical properties of the volatility data, as well as HP-filtered GDP data in figure 1 are discussed in a later section.

Our second calibration target in table 2 is the interest rate spread paid by financial firms. We associate the interest rate spread in the data with $R_g^d - R$ in our model. Loosely, we have in mind that R_g^d is the interest rate on the face of the loan contract. The 60 annual

Table 1. Baseline Model Parameter Values

| Meaning	Name	Value
Panel A: financial parameters		
Return parameter, bad entrepreneur	b	-0.09
Return parameter, good entrepreneur	g	0.00
Constant, effort function	$\bar{a}$	0.83
Slope, effort function	$\bar{b}$	0.30
Lump-sumtransfer fromhouseholds to bankers	$\tilde{T}$	0.38
Fraction of banker net worth that stays with bankers	γ	0.85
Panel B: Parameters that do not affect steady state		
Steady state inflation (APR)	$400(\pi - 1)$	2.40
Taylor rule weight on inflation	α_{π}	1.50
Taylor rule weight on output growth	$\alpha_{\Delta y}$	0.50
Smoothing parameter in Taylor rule	ρ_p	0.80
Curvature on investment adjustment costs	S''	5.00
Calvo sticky price parameter	ξ_p	0.75
Calvo sticky wage parameter	ξ_w	0.75
Panel C: Nonfinancial parameters		
Steady state gdp growth (APR)	μ_{z^*}	1.65
Steady state rate of decline in investment good price (APR)	Υ	1.69
Capital depreciation rate	δ	0.03
Production fixed cost	Φ	0.89
Capital share	α	0.40
Steady state markup, intermediate good producers	λ_f	1.20
Habit parameter	b_u	0.74
Household discount rate	$100(\beta^{-4} - 1)$	0.52
Steady state markup, workers	λ_w	1.05
Frisch labor supply elasticity	$1/\sigma_L$	1.00
Weight on labor disutility	Ψ_L	1.00
Steady state scaled government spending	$\tilde{g}$	0.89

Source: Authors' elaboration.

**Table 2: Steady State Calibration Targets for Baseline Model**

Variable meaning	Variable name	Magnitude
Cross-sectional standard deviation of quarterly non-financial firmequity returns	s^b	0.20
Financial firminterest rate spreads (APR)	$400(R_g^d - R)$	0.60
Financial firmleverage	L	20.00
Allocative efficiency of the banking system	$p(e)e^g + (1-p(e))e^b$	1
Profits of intermediate good producers (controled by fixed cost, Φ)		0
Government consumption relative to GDP (controlled by $\tilde{g}$)		0.20
Growth rate of per capita GDP (APR)	$400(\mu_z^* - 1)$	1.65
Rate of decline in real price of capital (APR)	$400(\Upsilon - 1)$	1.69

Source: Authors' elaboration.

Figure 1. Cross-section Standard Deviation Financial Firm Quarterly Return on Equity, HP-filtered U.S. Real GDP

Quarterly data

——— *Cross section volatility (left scale)*
- - - - - *HP filtered GDP (right scale)*
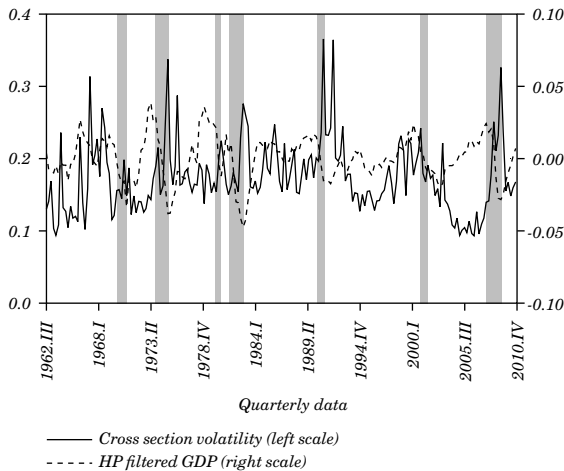
Source: Authors' calculations with data from Federal Reserve Board of Governors.

Figure 2. 3 Month U.S. Libor versus 3 Month T-bill (APR)
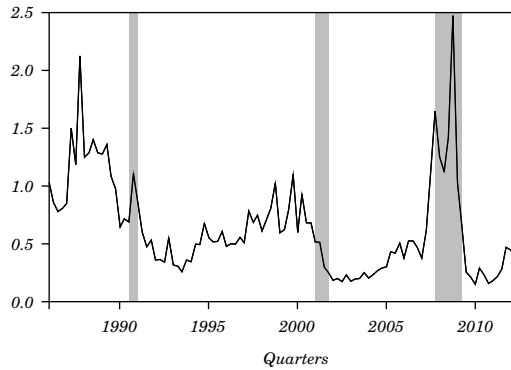
Quarters

Source: Federal Reserve Board of Governors.

basis point interest rate spread in table 2 is the sample average of the data on spreads in figure 2. That figure displays quarterly data on the spread on 3-month loans, measured by the London Interbank Offer Rate, over the rate on 3-month U.S. government securities. The data are reported in annual percent terms.

The third calibration target is leverage L, which we set to 20. We based this on sample leverage data reported in figure 3 of CGFS (2009). According to the results reported there, the leverage of large U.S. investment banks averaged around 25 since 1995 and the leverage of U.S. commercial banks averaged around 14 over the period.[16] Our value $L = 20$ is a rough average of the two.

For the remaining calibration targets we use the average growth of U.S. per capita GDP and the average decline in U.S. durable goods prices. We set the allocative efficiency of the financial system in steady state to unity. We suspect that this is in the nature of normalization. Finally, we set the fixed cost in the production function so that profits of the intermediate good firms in steady state are zero. We do not allow entry or exit of these firms, and the implausibility of this assumption is perhaps minimized with the zero steady state profit assumption.

16. The data of large U.S. investment banks are based on information about Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch and Morgan Stanley.

The parameters pertaining to the financial sector that remain to be determined are $\bar{b}$ and γ. The parameter, $\bar{b}$, is important in our analysis. If $\bar{b}$ is sufficiently low, then the unobserved and observed equilibria are similar, and the essential mechanism emphasized in this paper is absent. With low $\bar{b}$, our baseline model inherits the property of the observable effort equilibrium, that binding leverage reduces social welfare. If $\bar{b}$ is too high, then the incentive to exert effort is substantial and there ceases to exist an interior equilibrium with $p(e) < 1$ in the baseline model. We balance these two extremes by setting $\bar{b} = 0.3$. With $\bar{b} = 0.2$, social welfare falls when leverage is restricted by a very modest amount, to 19.999. The parameter γ resembles a similar object in Bernanke, Gertler and Gilchrist (1999), which assigns a value of 0.98 to it. We found that with such a large value of γ, the dynamic response of variables to a monetary policy shock is very different from the results based on vector auto regressions (VARs) reported in CEE. In particular, a jump in the monetary policy shock in (24) drives inflation and output up, rather than down. We are still exploring the economic reasons for this result. However, we noticed that with $\gamma = 0.85$, the impulse responses to a monetary policy shock appear more nearly in line with the results reported in CEE. This is why we chose the value $\gamma = 0.85$. We are investigating what the implications of micro data may be for the value of this parameter.

The parameters in panel B were assigned values that are standard in the literature. The steady state inflation rate corresponds roughly to the actual U.S. experience in recent decades. The Calvo sticky price and wage parameters imply that prices and wages, on average, remain unchanged for about a year. Similarly, the parameter values in panel C are also fairly standard.

3.2 The Steady State Effects of Leverage

We consider the impact on welfare and other variables of imposing a binding leverage restriction. The results are reported in table 3. The first column of numbers displays the steady state properties of our baseline model, the unobservable effort model without any leverage restrictions. In that model, the assets of the financial system are 20 times their net worth. The second column of numbers shows what happens to the steady state of the model when all parameters are held at their values in table 1, but a binding leverage restriction of 17 is imposed. The last two columns of numbers report the same

Table 3. Steady State Properties of the Model

Variable meaning	Variable name	Unobserved effort		Observed effort	
		Leverage restriction		Leverage restriction	
		Non-binding	Binding	Non-binding	Binding
Spread	$400(R_g^d - R)$	0.600	0.211	NA	NA
Multiplier on cash constraint	ν	0.060	0.040	0.000	0.000
Scaled consumption	c	1.840	1.880	2.010	1.950
Scaled GDP	y	4.430	4.370	4.680	4.430
Labor	h	1.180	1.160	1.150	1.140
Scaled capital stock	k	51.520	51.400	59.750	53.860
Capital output ratio	$k/(c + i + g)$	11.630	11.750	12.780	12.150
Bank assets	$N + d$	51.520	51.310	59.550	53.680
Bank net worth	N	2.580	3.020	2.580	3.160
Bank deposits	d	48.940	48.290	56.980	50.520
Bank leverage	$(N+d)/N$	20.000	17.000	23.120	17.000

Table 3. (continued)

| | | Unobserved effort | | Observed effort | |
| | | Leverage restriction | | Leverage restriction | |
Variable meaning	Variable name	Non-binding	Binding	Non-binding	Binding
Bank return on equity (APR)	$400\left(\frac{[p(e_t)R^g_{t+1}+(1-p(e_t))R^b_{t+1}](N_t+d_t)-R_t d_t}{N_t}-1\right)$	4.590	14.960	4.590	17.630
Equity portion of bank return (APR)	$400(p(e_t)R^g_{t+1}+(1-p(e_t))R^b_{t+1}-1)$	4.590	5.200	4.590	5.360
Deposit portion of bank return (APR)	$400[p(e_t)R^g_{t+1}+(1-p(e_t))R^b_{t+1}-R_t]\frac{d_t}{N_t}$	0.000	9.760	0.000	12.270
Benchmark return on capital (APR)	$400(R^k-1)$	4.590	4.470	3.230	4.000
Bank efficiency	$p(e)e^g+(1-p(e))e^b$	1.000	1.002	1.003	1.003
Fraction of firms with good balance sheets	$p(e)$	0.962	0.982	1.000	1.000
Benefit of leverage (in c units)	100χ	NA	1.190	NA	-2.700
Benefit of making effort observable (in c units)	100χ	NA	NA	6.110	2.030

Source: Authors' calculations.
Note: (i) NA, not applicable, indicates that the number is not defined. (ii) All calculations based on a single set of parameter values, reported in table 1.

results as in the first two columns, but they apply to the version of our model in which effort is observable. We first consider the results for the unobserved effort version of the model.

When leverage restrictions are imposed, table 3 indicates that bank borrowing d declines. A consequence of this is that the interest rate spread on banks falls. To gain intuition into this result, we can see, from the fact that the multiplier, ν, on the cash constraint (3) is positive, that the cash constraint is binding (for ν, see (9)). This means that the creditors of banks with poorly performing assets must share in the losses, i.e., R_g^d is low. However, given the zero profit condition of mutual funds (1), it follows that R_b^d must be high. That is, $R_b^d > R$ and $R_g^d > R$. We can see from (3) that, given R^b and bank net worth, creditors of ex post bad banks suffer fewer losses the smaller are their deposits. This is why the value of R_b^d that solves (3) with equality increases with lower deposits. This in turn implies, via the mutual funds' zero profit condition that R_g^d falls towards R as d falls. Thus, deposit rates fluctuate less with the performance of bank portfolios with smaller d. This explains why the interest rate spread falls from 60 basis points in the baseline model to 21 basis points with the imposition of the leverage restriction. A closely related result is that ν falls with the introduction of the binding leverage constraint.

The reduction in the interest rate spread faced by banks helps to improve the efficiency of the economy by giving banks an incentive to increase e (see (8)). But these effects alone only go part way in explaining the full impact of imposing a leverage restriction on this economy. There is also an important general equilibrium, a dynamic effect of the leverage restriction that operates via its impact on banker net worth.

To understand this general equilibrium effect, we observe that a leverage restriction, in effect, allows banks to collude and behave like monopsonists. Deposits are a key input for banks and unregulated competition drives the profits that banks earn on deposits to zero. We can see this from the d equation in (9). That equation shows that in an unregulated banking system, the profits earned by issuing deposits are zero in expectation. This zero profit condition crucially depends on banks being able to expand deposits in case they earn positive profits on them. When a binding leverage restriction is imposed, this competitive mechanism is short-circuited. The d equation in (9) is replaced by (17), where $\Lambda \geq 0$ is the multiplier on the leverage constraint in the banker problem. When this multiplier is positive the bankers make positive profits on deposits. To explain this further, it

is useful to focus on a particular decomposition of the rate of return on equity for banks. This rate of return is:

$$\frac{\left[p\left(e_t \right) R^g_{t+1} + \left(1 - p\left(e_t \right) \right) R^b_{t+1} \right] N_t}{N_t}$$

$$\frac{+ \left[p\left(e_t \right) \left(R^g_{t+1} - R^d_{g,t+1} \right) + \left(1 - p\left(e_t \right) \right) \left(R^b_{t+1} - R^d_{b,t+1} \right) \right] d_t}{N_t} - 1$$

$$= \overbrace{p\left(e_t \right) R^g_{t+1} + \left(1 - p\left(e_t \right) \right) R^b_{t+1} - 1}^{\text{equity portion of bank rate of return on bank equity}}$$

$$\underbrace{+ \left[p\left(e_t \right) \left(R^g_{t+1} - R^d_{g,t+1} \right) + \left(1 - p\left(e_t \right) \right) \left(R^b_{t+1} - R^d_{b,t+1} \right) \right] \frac{d_t}{N_t}}_{\text{deposit contribution to rate of return on bank equity}}$$

These three objects are displayed in table 3, after substituting out for $R^d_{g,t+1}$ and $R^d_{b,t+1}$ using the mutual fund zero profit condition. The d equation in (9) implies that, in steady state, the object in square brackets in the deposit contribution to banks' return on equity is zero.[17] So, the fact that d_t/N_t is very large when leverage is 20 has no implication for bank profits. However, with the imposition of the leverage restriction, the object in square brackets becomes positive and then the large size of d_t/N_t is very important. Indeed, it jumps from 0 to 9.76 (APR) when the leverage restriction is imposed. This is the primary reason why banks' rate of return on equity jumps from only 4.59 percent per year in the absence of regulations to a very large 14.96 percent per year when the leverage restriction is imposed. A small additional factor behind this jump is that the equity portion of bankers' rate of return on equity jumps a little too. That reflects the improvement in the efficiency of the banking system as e rises with the imposition of the leverage regulation. To see this, recall from (26) that the gross return on bank assets is given by:

$$p(e)R^g + (1 - p(e)) - R^b \qquad (29)$$

$$= \left[p(e)e^g + (1 - p(e)) e^b \right] R^k.$$

17. Here, we also use the mutual fund, zero-profit condition.

From this we see that the gross return on bank assets can rise, even if R^k falls a little, if the allocative efficiency of the banking system improves enough.[18]

With the high rate of profit it is not surprising that in the new steady state associated with a leverage restriction, bank net worth is higher. Indeed, it is a substantial 17 percent higher. This effect on bank net worth mitigates one of the negative consequences of the leverage restriction. We can see this from (8), which shows that banker effort is not just decreasing with an increased spread between R_b^d and R_g^d, but it is also a function of the total quantity of assets under management. Thus, the bank profits occasioned by the imposition of leverage restrictions raise banker net worth and mitigate the negative impact on banker efficiency of a fall in deposits.

As a way of summarizing the results in table 3 for the unobserved effort model of this section, we examine the impact of leverage on welfare. We suppose that the social welfare function is given by:

$$u = \log\left(c - \frac{b}{\mu_{z^*}}c\right) - \frac{\psi_L}{1 + \sigma_L}h^{1+\sigma_L} - \frac{1}{2}e^2,$$

where c represents C_t/z_t^* in steady state. Let u^l and u^{nl} denote the value of this function in the equilibrium with leverage imposed and not imposed, respectively. Let $u^{nl}(\chi)$ denote utility in the equilibrium without leverage in which consumption c^{nl} is replaced by $(1 + \chi)c^{nl}$. We measure the utility improvement from imposing leverage by the value of χ that solves $u^{nl}(\chi) = u^l$. That is,

$$\chi = e^{u^l - u^{nl}} - 1.$$

In the table we report 100χ. Note that the welfare improvement from imposing leverage is a very substantial 1.19 percent. We suspect that, if anything, this understates the welfare improvement somewhat. According to the table, the quantity of capital falls a small amount with the imposition of the leverage restriction while the efficiency of the banking system improves. This suggests that during the transition between steady states (which is ignored in our welfare

18. The rate of return R^k on capital falls somewhat because the capital to labor ratio rises, and this reduces the rental rate of capital. This is the only input into R^k that changes with the imposition of leverage.

calculations), investment must be relatively low and consumption correspondingly high.

We now discuss the last two columns in table 3. The column headed "non-binding" describes properties of the equilibrium of our model when effort is observable and the model parameters take on the values in table 1. The column headed "binding" indicates the equilibrium when leverage is restricted to 17. We do not report interest rate spreads for the observable effort model because, as indicated above, spreads are not uniquely determined in that model. Comparing the results in the last two columns with the results in the first two columns allows us to highlight the central role in our analysis played by the assumption that effort is not observable. The welfare results in the table provide two ways to summarize the results.

First, note that imposing a leverage restriction on the model when effort is observed implies a very substantial 2.70 percent drop in welfare.[19] Evidently, leverage restrictions are counterproductive when effort is observable. Second, the results indicate that the lack of observability of effort implies a substantial reduction in welfare. In the absence of a leverage restriction, the welfare gain from making effort observable is 6.11 percent.[20] When a binding leverage limit of 17 is in place, then the welfare gain from making effort observable is also a substantial 2.03 percent.[21]

We now discuss why it is that the observable effort equilibrium is so much better than the equilibrium in which effort is not observable. We then sum up by pointing out the benefits of the leverage restriction on the unobserved effort economy explaining what it is about the leverage restriction that improves welfare.

Making effort observable results in higher consumption and output, and lower employment. These additions to utility are partially offset by the utility cost of extra effort by bankers. This extra effort by bankers in the observable effort equilibrium is the key to understanding why consumption and capital are higher, and

19. The simultaneous drop in the capital stock and the absence of any change in the efficiency of the banking system suggests that when the transition is taken into account, the drop in welfare may be smaller.

20. It is not clear how taking into account the transition between steady states would affect this welfare calculation. In the steady state with observable effort, the quantity of capital is higher but the efficiency of the banking system is also greater. The impact of the transition on welfare depends on the extent to which the higher amount of capital reflects increased efficiency and/or a reduction in consumption during the transition.

21. The observations about the impact of the transition on welfare calculations made in the previous footnote apply here as well.

labor lower, in that equilibrium. To see this, note that the steady state version of (6), combined with (29), imply:

$$R = [p(e) \, e^g + (1 - p(e)) \, e^b] \, R^k.$$

When e rises with observability of effort, the object in square brackets (the allocative efficiency of the banking system) increases and, absent a change in R^k, would cause a rise in R. Imagine that that rise in R did occur, stimulating more deposits. That would lead to more capital, thus driving R^k down. In the new steady state, R is the same as it was before effort was made observable. Thus, across steady states R^k must fall by the same amount that the efficiency of the banking system rises. The fall in R^k implies a rise in the capital to labor ratio k/h. According to table 3, this rise is accomplished in part by an increase in k and in part by a decrease in h. The higher steady state capital is sustained by higher intermediation $N + d$ and this primarily reflects a higher level of deposits.[22] Imposing the leverage restriction on the unobserved effort economy moves consumption, employment and effort in the same direction that making effort observable does. This is why imposing the leverage restriction raises welfare.

3.3 Dynamic Properties of the Model

In this section we consider the dynamic effects of a monetary policy shock and four financial shocks.

3.3.1 Monetary policy shock

Figure 3 displays the responses in our baseline model to a 25 basis point shock to monetary policy. First, consider the standard macroeconomic variables. The shock has a persistent, hump-shaped and long-lived effect on output, consumption and investment. The maximal decline of 0.35 and 0.55 percentage points, respectively, in GDP and investment occurs after about two years. In the case of consumption, the maximal decline occurs three years after the shock and the maximal decline is a little over 0.35 percent. Inflation drops a modest 8 annualized basis points. Unlike the pattern reported in CEE, the response in inflation does not display a hump-shape. However,

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22. In the case with no leverage restriction, the rise in $N + d$ is entirely due to a rise in d.

Figure 3. Dynamic Response of Baseline Model to Monetary Policy Shock

GDP

Consumption

Investment

Inflation (APR)

Risk free rate (APR)

Interest rate spread (APR)

Prob of success

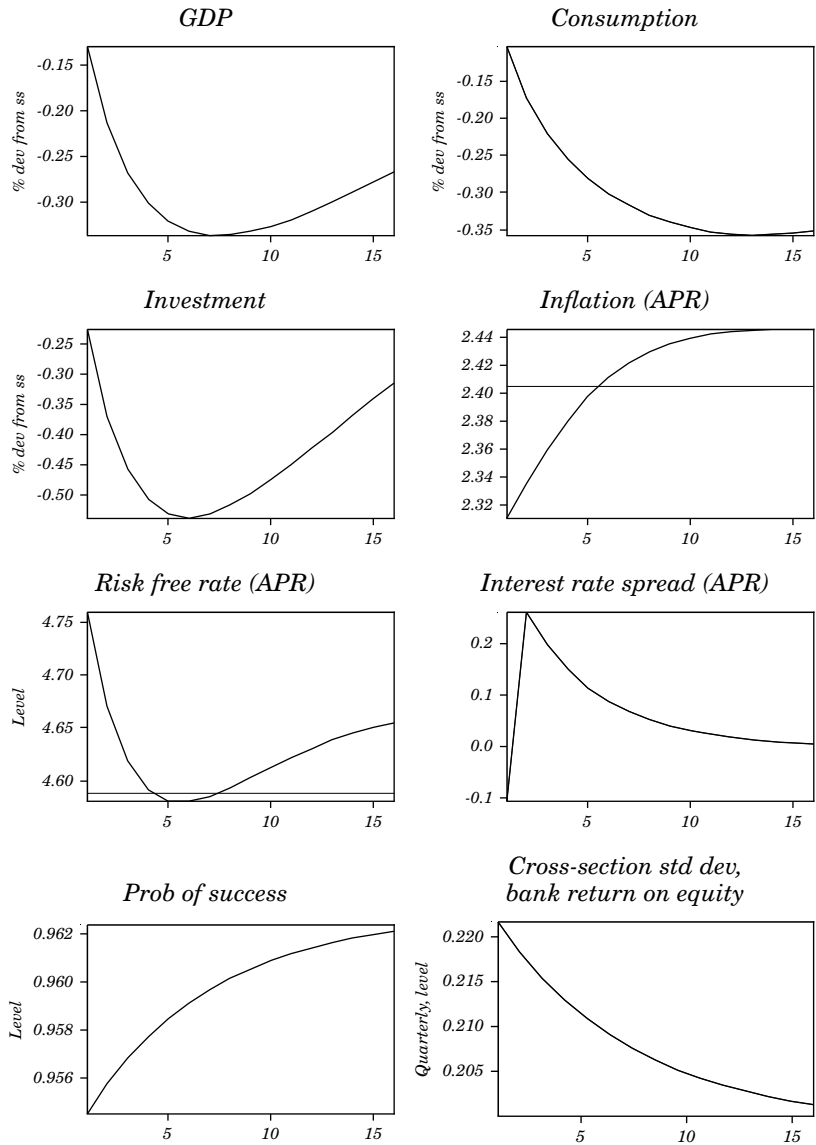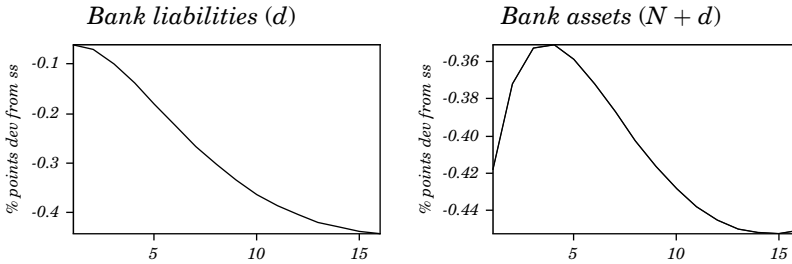Cross-section std dev, bank return on equity

Figure 3. (continued)

Source: Authors' elaboration.

direct comparison between the results in figure 3 and VAR-based estimates of the effects of monetary policy shocks reported in CEE and other places is not possible. The latter estimates often assume that aggregate measures of economic activity and prices and wages are predetermined within the quarter to a monetary policy shock. In our model, this identifying assumption is not satisfied. One way to see this is to note that the actual rise in the interest rate is only 15 basis points in the period of the shock. The fact that the interest rate does not rise the full 25 basis points of the policy shock reflects the immediate negative impact on the interest rate of the fall in output and inflation. Still, it seems like a generally positive feature of the model that the implied impulse responses correspond, in a rough qualitative sense, to the implications of VAR studies for aggregate variables and inflation.

Now, consider the impact on financial variables. The reduction in output and investment reduces R^k by two channels: it reduces the rental rate of capital and the value of capital $P_{k'}$. Both of these have the effect of reducing bank net worth. The reduction in bank net worth leads to a tightening of the cash constraint (3). The result is that the interest rate spread on banks increases and banker effort declines. That is, $p(e)$ falls 70 basis points. This in turn is manifest in a rise in the cross-sectional dispersion of bank equity returns. Interestingly, cross-sectional dispersion in the rate of return on financial firm equity is countercyclical in the data (figure 1). Finally, bank assets $N + d$ and bank liabilities d both decline.

The relative size of the decline in $N + d$ and d is of some interest. To pursue this, it is useful to focus on a particular decomposition of the percent change in bank leverage. Let Δx denote $(x - x^s)/x^s$,

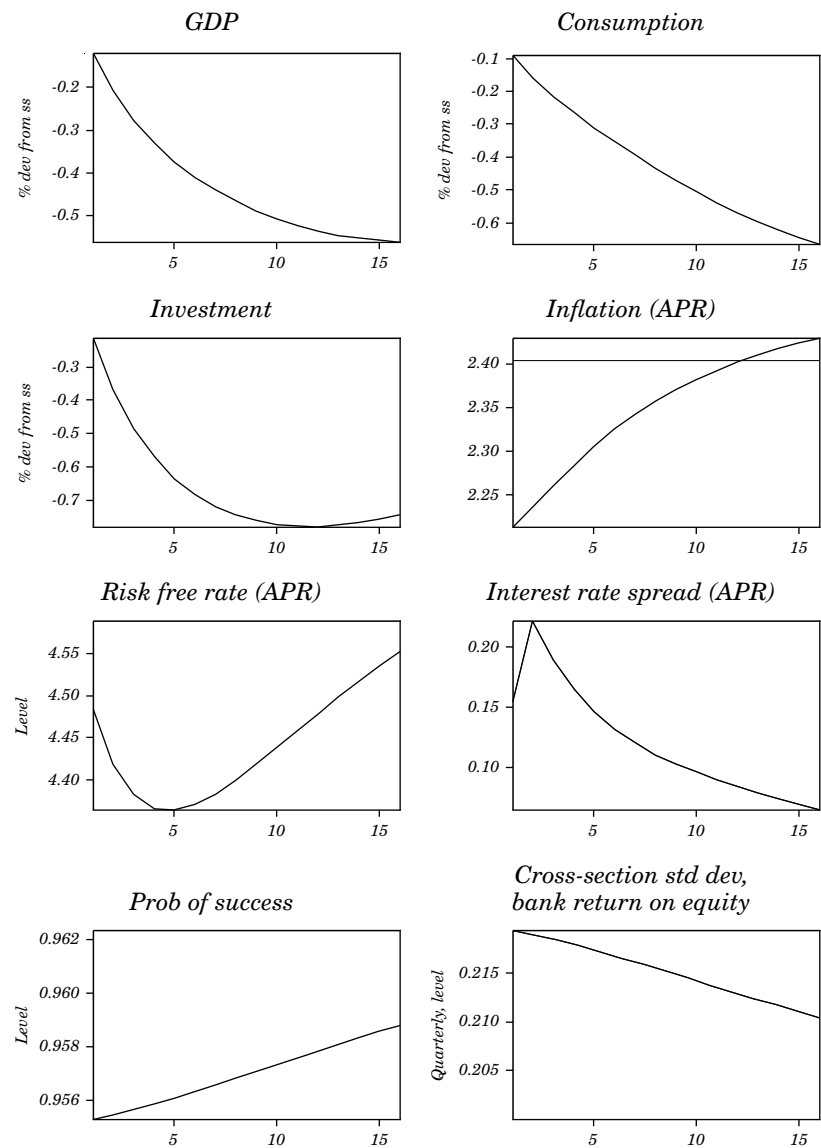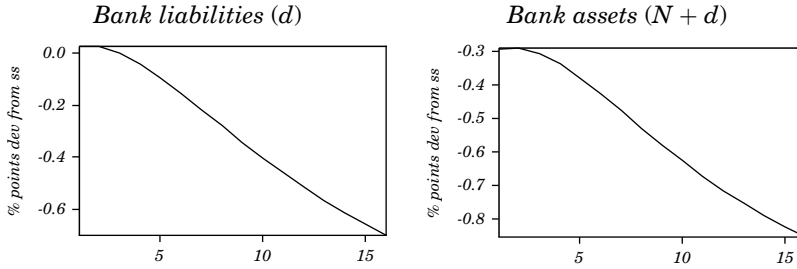**Figure 4. Dynamic Response of Baseline Model to ϒ Shock**

GDP

Consumption

Investment

Inflation (APR)

Risk free rate (APR)

Interest rate spread (APR)

Prob of success

Cross-section std dev, bank return on equity

Figure 4. (continued)

Bank liabilities (*d*)   *Bank assets* (*N* + *d*)

Source: Authors' elaboration.

where x^s is a reference value (perhaps its lagged value) of a variable x. Then, letting L denote bank leverage $(N + d)/N$ we have[23]

$$\Delta L = (L - 1)[\Delta d - \Delta (N + d)].$$

Using this expression we can infer from figure 3 that our model implies a rise in leverage in the wake of a monetary-policy induced contraction. Recent literature suggests this implication is counterfactual (see Adrian, Cola and Shin, 2012). We suspect that a version of the model could be constructed in which credit responds more and net worth less, so that leverage is pro-cyclical.

3.3.2 Financial shocks

The dynamic responses of the model variables to our three financial shocks are displayed in figures 4, 5 and 6. A notable feature of these figures is how similar they are, at least qualitatively. In each case, consumption, investment, output, inflation and the risk-free rate all fall in response to the shock. The interest rate spread rises and the cross-sectional dispersion in bank equity returns jumps as $p(e)$ falls. Finally, bank assets and liabilities both fall. However, the former fall by a greater percent, so that leverage is countercyclical in each case. It is perhaps not surprising that the risk shock has the greatest quantitative impact on $p(e)$.

23. Note that $\Delta(N + d) = N/(N + d) \ \Delta N + d/(N + d)\Delta d$, so that $\Delta N = (N + d)/\Delta(N + d) - d/N\Delta d$. Also, $\Delta L = \Delta(N + d) - \Delta N$.
   The formula in the text follows by substituting out for ΔN from the first expression.

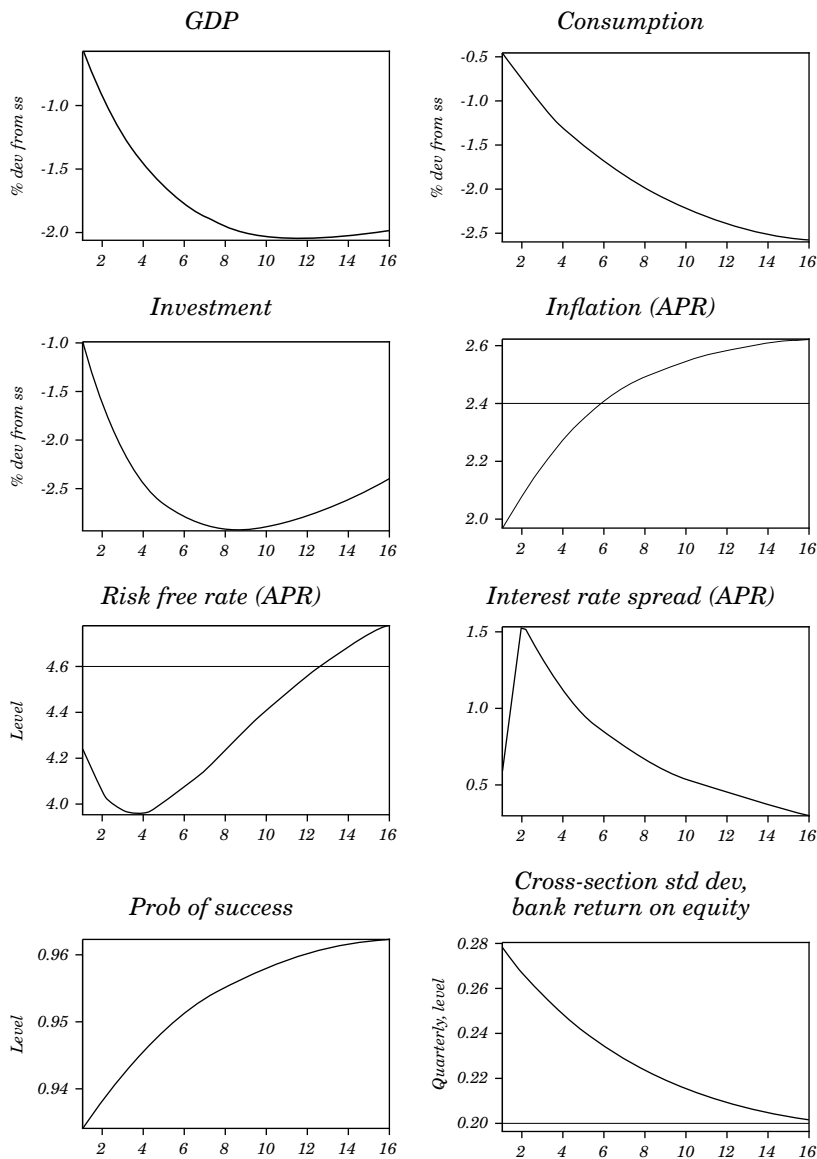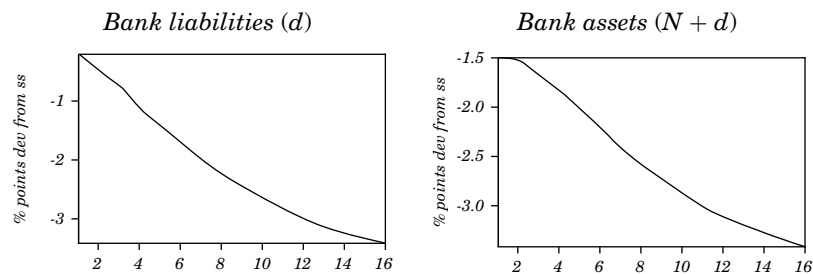**Figure 5. Dynamic Response of Baseline Model to Risk Shock**

Figure 5. (continued)

Bank liabilities (*d*)

Bank assets (*N* + *d*)

Source: Authors' elaboration.

Figure 6. Dynamic Response of Baseline Model to *T* Shock

GDP

Consumption

Investment

Inflation (APR)

Figure 6. (continued)

Risk free rate (APR)

Interest rate spread (APR)

Prob of success

Cross-section std dev, bank return on equity

Bank liabilities (d)

Bank assets ($N + d$)
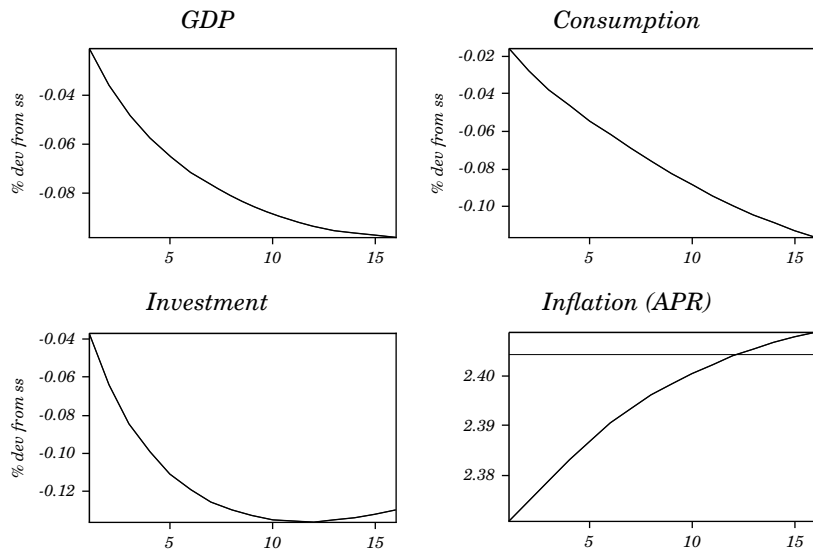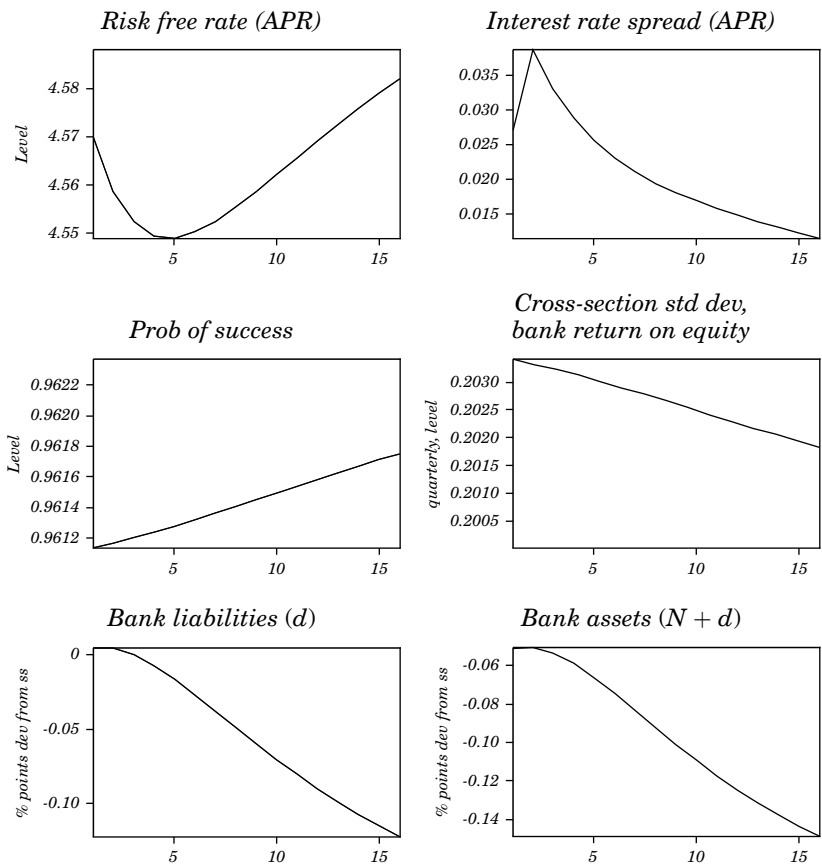
Source: Authors' elaboration.

4. CONCLUSION

Bank leverage has received considerable attention in recent years. Several questions have been raised about leverage:
— Should bank leverage be restricted, and how should those restrictions be varied over the business cycle?
— How should monetary policy react to bank leverage, if at all?

This paper describes an environment that can in principle be used to shed light on these questions. We have presented some preliminary results by studying the implications for leverage in steady state. We showed that steady state welfare improves substantially with a binding welfare restriction. There are several ways to understand the economics of this result. We pursue one way in this paper. Bigio (2012) takes an alternative approach, in which he relates the improvement in welfare to the operation of a pecuniary externality. Either way, leverage restrictions help to correct a problem in the private economy. For this reason, we think the model environment is an interesting one for studying the questions listed above.

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